Optimal Inheritance Taxation with Endogenous Wealth Inequality

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This draft: May 2014

Abstract

We extend the model of Piketty and Saez (2013) by endogenizing the wealth distribution, such that the optimal tax formular incorporates the distributional feedback of aggregate bequest elasticities to tax rates. The optimal tax formular demonstrates clearly the efficiency-equity tradeoffs caused by inheritance taxation, and the results feature positive optimal inheritance tax rates.

1 Introduction

We extend the model of Piketty and Saez (2013) by endogenizing the wealth distribution, such that the optimal tax formular incorporates the distributional feedback of aggregate bequest elasticities to tax rates. The optimal tax formular demonstrate clearly the efficiency-equity tradeoffs caused by inheritance taxation, and the results feature positive optimal inheritance tax rates.

The population has heterogeneous "joy-of-giving" bequest motives, supplies labor inelastically and has productivity shocks. After the bequest motives and labor productivities are fully realized, households make optimal resource allocation decisions between consumption and bequest. The government collects inheritance taxes and redistributes the tax revenue. The government maximizes total welfare of the top households, and also values the redistribution of tax revenue explicitly.

2 Inheritance taxation with inelastic labor supply

In this section we build a simple model of estate taxation. To explicitly focus on the wealth accumulation process across generations, we analyze a one-period overlapping generation model with heterogeneous bequest motives. For simplicity, we assume that labor is supplied inelastically, and we only consider partial equilibrium. Each household consists of only one agent, whos lives for one period and is followed by one offspring. At the beginning of period t, household i draws the idiosyncratic labor productivity w_{ti} and bequest motive χ_{ti} from i.i.d. distributions $\{w_{ti}\}$ and $\{\chi_{ti}\}$. To isolate the effect of estate taxation in wealth accumulation dynamics, we assume that household i's capitalised bequest received is subject to estate taxation, but labor income and government transfer is not taxed. As a result, household i's total wealth is then consist of the after-tax capitalised inheritance from the previous generation, labor income and lump sum transfers from the government. Households then make optimal decisions about consumption and bequest saved for the next generation.

Literature has no consensus regarding the presense and significance of bequest motives. Kopczuk and Lupton (2007) has documented that approximately three-fourths of elderly single population has a bequest motive after analysing data from AHEAD; Hendricks (2007) has found that 70% of the U.S. households receive no bequests, and inheritances are roughly as concentrated as wealth from the data of SCF and PSID; Arrondel and Masson (2006, "Altruism, exchang or indirect reciprocity: what do the data on family transfers show?") provided a survey of the literature on bequest motives. The disparity among empirical evidences of bequest motives can be attributed to the fact that researchers cannot quantify the importance of the intentional/altruistic bequests from the precautionary/accidental terminal wealth. Generally, the strand of literature that features no bequest motives has difficulties matching the strongly right-skewed wealth distribution, and models with altruism cannot explain the enoumous wealth concentrated at the top 1%. These empirical findings motivate us to model the heterogeneity of bequest motives explicitly.

In this paper, we sideline all the life cycle component of wealth accumulation and assume that every household has a "joy-of-givng" bequest motive, i.e. $\chi_{ti} > 0$; and for simplicity, we assume $\{\chi_{ti}\}$ is i.i.d. Furthermore, our model also features a discrete "operational" bequeathing pattern where only the households with sufficient wealth leave positive bequest. Specifically, household ti has concave utility $\frac{u(c_{ti},b_{t+1i})^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$, where $u\left(c_{ti}, b_{t+1i}\right)$ is in CES form with coefficient $0 \leq \alpha < 1$. Household utility maximization problem is:

$$\max_{c_{ti},b_{t+1i}} \left[c_{ti}^{\alpha} + \chi_{ti}\left(\left(1-\tau\right)\left(b_{t+1i}+\theta\right)\right)^{\alpha}\right]^{\frac{1-\gamma}{\alpha}} \text{ s.t.}$$

$$c_{ti} + b_{t+1i} = R_t b_{ti}\left(1-\tau\right) + w_{ti} + T_t$$

Note here the household heterogeneity is two-dimension: household receives different inheritance from the previous generation, and also has different wage earning ability. Solving for the bequest intended for the next generation gives:

$$b_{t+1i} = \begin{cases} B_{ti}\left(R_t b_{ti}\left(1-\tau\right) + w_{ti} + T_t - \hat{\theta}_{ti}\right) \text{ if } A_{ti} > \hat{\theta}_{ti} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

where $B_{ti} = \left(1 + \chi_{ti}^{\frac{1}{\alpha-1}} (1-\tau)^{\frac{\alpha}{\alpha-1}}\right)^{-1}$ is the household specific bequest propensity, $\hat{\theta}_{ti} = \chi_{ti}^{\frac{1}{\alpha-1}} (1-\tau)^{\frac{\alpha}{\alpha-1}} \theta$ is the wealth threshold for bequest and $A_{ti} \equiv R_t b_{ti} (1-\tau) + w_{ti} + T_t$ is lifetime household wealth. Three points are worth noting. First, households with stronger bequest motives leave larger proportion of their wealth as bequest. Second, we assume that every household possesses innate altruistic motive, but households with low wealth leaves zero bequest. In other words, bequest motive is only operational when wealth A_{ti} exceeds the threshold $\hat{\theta}_{ti}$. This discrete pattern of bequest explains the phenomenon where families at the lower end of wealth distribution seldom leave bequests. Third, the wealth threshold $\hat{\theta}_{ti}$ is household specific and positively correlated with the household bequest propensity, which implies that, more altruistic households not only are more likely to leave bequests, but also leave proportionally more.

More importantly, higher estate tax raises the relative price of bequest and diminishes the "joy-of-giving" from bequeathing. For the bequest leavers, bequest propensity B_{ti} is a decreasing function of estate tax τ. As higher estate tax raises the relative price of bequest, households would adjust the consumption to bequest ratio and scale down the proportion for bequest. Moreover, the depressed bequest propensity goes hand in hand with higher wealth threshold. As higher estate taxation depreciates the utility from leaving bequest, it also swings more households from leaving positive bequest to zero bequest on the margin. As a result, higher estate tax discourages bequest. It is useful to consider the case where CES coefficient α converges to zero. In such limit case, household optimal bequest decisions are independent of estate taxation, and estate tax no longer entails distortion to the intertemporal resource allocations.

On the other hand, for the zero bequest receiver, household wealth depends solely on labor income and government transfers: $A_{ti} = w_{ti} + T_t$. Generally, zero bequest receivers have lower household wealth, and are more likely to leave zero bequest as well. For the household receives no bequest, the escape is through labor income. Accordingly, we make the following "American dream" assumption:

Assumption 1 $Prob\left(w_{ti} + T > \hat{\theta}_{ti}\right) > 0$

Assumption (1) ensures that, although parents who received zero bequest are more likely to leave zero bequest as well, their children can escape the "vicious cycle" by earning higher labor income. This assumption accords with the spirit of "hard work" and the promise of "the American dream". Technically this assumption is to guarantee that zero bequest is not an absorbing state in the bequest accumulation process; it is crucial for the convergence of endogenous wealth distribution.

For the households that leave positive bequest, the wealth accumulation process is given by

$$b_{t+1i} = B_{ti} R_t (1-\tau) b_{ti} + B_{ti} \left(w_{ti} + T_t - \hat{\theta}_{ti}\right) \tag{2}$$

For constant gross interest rate and government transfers, and under additional regulative conditions, the stochastic process (2) generates an unique and stationary distribution of wealth $\{b_t\}$. The distribution has asympototic Pareto tail with tail index defined by

$$E\left[BR\left(1-\tau\right)\right]^{\lambda}=1 \tag{3}$$

Note that from equation (1) we know that distribution of bequest $\{b_{ti}\}$ and wealth $\{A_{ti}\}$ have the same asymptotic tail index; that is, at the top 1%, bequest is as concentrated as wealth.

To formally show the existence and uniqueness of tail index λ in equation (3), we need to make the following assumptions on the underlying stochastic process of $\{w_n\}$ and $\{\chi_n\}$. Define

$$\rho\left(\chi_n\right)\equiv B\left(\chi_n\right)R\left(1-\tau\right) \text{ and } \omega\left(\chi_n,w_n\right)\equiv B\left(\chi_n\right)\left(w_n+T-\hat{\theta}_n\left(\chi_n\right)\right)$$

then equation (2) can be rewritten as the standard stochastic different equation

$$b_{n+1}=\rho_n b_n+\omega_n$$

Theorem 2 *Assume $\{\chi_n,\omega_n\}_n$ is i.i.d. If the following conditions are satisfied:*

- *There exists $0<\lambda<1$ such that $E\left(\rho\right)^{\lambda}=1$;*
- *$\omega_i\geq 0$ for all i, and $\omega_i>0$ for some i;*
- *There exists $\delta>0$ such that $E\left(\omega\right)^{\lambda+\delta}<\infty$*

then the tail of the stationary distribution of b_n, $\Pr\left(b_n>b\right)$, is asymptotic to a Pareto law

$$\Pr\left(b_n>b\right)\sim kb^{-\lambda}$$

where $\lambda>1$ is given by

$$E\left(\rho\right)^{\lambda}=1 \tag{4}$$

Proof. See Appendix. ■

Proposition 3 *The asymptotic Pareto tail index λ increases with higher inheritance tax rate τ.*

It can be shown that tail index λ increases with tax τ, in other words, higher estate tax τ implies lower top wealth inequality. Since both bequest propensity B and after-tax capital return $R\left(1-\tau\right)$ are decreasing with respect to tax τ, $\rho=RB\left(1-\tau\right)$ is an increasing function of τ, and we prove in the appendix that smaller ρ leads to larger λ through equation (4). As ρ measures dynamic link across generations, ρ being decreasing with τ shows higher estate tax dismays households' altruistic behaviors and weakens wealth linkage across generations. As a result, higher estate tax suppresses the wealth accumulation across generations, and lowers the inequality among top wealth households in the steady state. In the Appendix we rigorously prove Proposition 3, here we use a special case to demonstrate the monotone relationship between λ and τ.

2.1 Special case: Cobb-Douglas utility and inverse Pareto distributed $\{B_{ti}\}$

We assume household utility is Cobb-Douglas and bequest propensity distribution is inverse Pareto. Under the Cobb-Douglas utility, bequest propensity $B_{ti} = \frac{1}{1+\chi_{ti}^{-1}}$ only depends on the strength of bequest motive. In other words, estate tax no longer entails distortions to household resource allocation, but only depresses intertemporal wealth accumulation. We assume bequest propensity is inversely Pareto distributed, that is, $B = \frac{1}{x}$ and $x \sim Pareto\,(1, a)$, such that bequest propensity is strictly between zero and one. Furthermore, we assume that $(1-\tau)\,R > 1$ and $E\,(BR) < 1$. Then $E\left[B\,(1-\tau)\,R\right]^{\lambda} = 1$ implies

$$((1-\tau)\,R)^{\lambda} = \frac{\lambda}{a} + 1 \tag{4a}$$

Function $y = ((1-\tau)\,R)^{\lambda}$ and $h = \frac{\lambda}{a} + 1$ have only one positive interception. As higher inheritance tax τ rotates the function y towards x-axis, equation (4a) produces a larger interception point λ. The Pareto index λ is monotonely increasing in estate tax τ, confirming the intuition that higher estate estate dampens the wealth accumulation process and lower the top wealth inequality. This representation of Pareto index λ also make available more detailed comparative statistics.

Firstly, λ decreases if the bequest propensity is less concentrated towards zero, i.e. a increases. Since larger a implies the inverse of bequest propensity is less skewed to the right, and equivalently bequest propensity is more concentrated towards one, which increases the chance of consecutive highly altruistic draws of bequest propensity. Since rich dynasty climbs the wealth ladder by accumulating inheritance, as bequest propensity becomes more persistent, it is easier for wealth to keep rolling. Accordingly, the top wealth inequality will increase.

Secondly, λ decreases if gross capital return R is larger. If R is larger, capitalization of inheritance will play a more important role in wealth composition, and wealthier households benefits disproportionally more from both receiving and leaving bequest. Moreover, as larger R strengthens the dynamic link between generations, wealth statues becomes more persistent across generations and top wealth inequality would rise. It can be shown that the channel through which larger gross capital return intensifies top wealth inequality is robust under serial correlated capital return process (see Benhabib, Bisin and Zhu (2011)).

Lemma 4 *(1) Given fixed τ and R, if shape parameter $a_1 < a_2$, then $\lambda_1 > \lambda_2$ and $\frac{d\lambda_2}{d\tau} > \frac{d\lambda_1}{d\tau} > 0$, where λ_1 and λ_2 are from equation (*). (2) Given fixed τ and a, if gross capital return $R_1 < R_2$, then $\lambda_1 > \lambda_2$ and $\frac{d\lambda_1}{d\tau} > \frac{d\lambda_2}{d\tau} > 0$, where λ_1 and λ_2 are from equation (*).*

Proof. See graph (?) in appendix. ■

Lemma 3 confirms our intuition that larger a and R lowers λ and increase top wealth inequality. Moreover, larger a raises the sensitivity of λ with respect to tax rate τ, while larger R lowers it. This finding will have more implications in later analysis.

It is important to bear in mind that our discussion has been focused on top wealth accumulation process. Labor income has no impact on the tail index nor top inequality. Intuitively, the richer the household, the smaller share labor income accounts for household wealth composition, and more inconsequential is single-period labor income in top wealth ranking.

2.2 Steady state top tax revenue maximization

In this subsection we derive the optimal estate tax rate that maximizes the steady state tax revenue. We assume the government imposes a constant linear estate tax on the rich households, who leave a bequest more than a fixed cutoff level b^{*}[1], and we normalize without loss of generality the population of eligible rich households to unity. Note here we only consider the efficiency loss from taxation, as revenue collection per se does not explicitlly reward equality. The government's objective can be represented by

$$\max_{\tau} \tau R \int_{b^*}^{\infty} b_i di$$

As estate tax τ increases from zero to unity, the tail index λ should increase monotonely to infinity. In such extreme case, an exogenously fixed cutoff value b^* would be inappropriate and infeasible. But as we only consider the senarios where endogenous tail index is within the range of one and two, we think it is reasonable to assume the distribution above b^* is asymptotically Pareto. Therefore, for a sufficiently large cutoff bequest level b^*, the above objective function can be rewritten as:

$$\tau R \int_{b^*}^{\infty} b_i = \tau R \left(\frac{b^* \lambda(\tau)}{\lambda(\tau) - 1} \right) \tag{5}$$

Saez (2001) provides a similar formular for revenue maximizing tax rate. However, Saez (2001) does not consider the distributional response of tax base caused by changing tax rate. Equation (5) exactly incorporates the feedback from endogenous bequest distribution and decomposes the effects of higher estate tax into two channels. First, higher τ will raise total tax revenue, this is the direct *mechanical effect*. On the other hand, the term in the bracket exactly captures the distortion of tax on the tax base. From Proposition 3 we know that $\lambda(\tau)$ is increasing with τ, making total tax base $\frac{b^* \lambda(\tau)}{\lambda(\tau)-1}$ a decreasing function with respect to τ. This formular demonstrates the efficiency loss due to inheritance taxation. As higher τ lowers top wealth inequality and attenuates wealth

[1] We focus on cases where the asymptotic Pareto index $1 < \lambda(\tau) < 2$, such that the disccusion is empirically interesting.

concentration at the top, it also trims down the aggregate bequest eligible for taxation, thus creating a negative *behavioral response* from shrinking tax base. Therefore, the tax revenue maximization problem is simplified into

$$\max_{\tau} f(\tau) = \frac{\tau\lambda(\tau)}{\lambda(\tau)-1}$$

The first order condition implies that the tax revenue maximizing tax rate τ satisfies the following condition:

$$\frac{\lambda(\tau)}{\lambda(\tau)-1} - \tau\frac{\lambda'(\tau)}{(\lambda(\tau)-1)^2} = 0 \tag{6}$$

Note that the tail index $\lambda(\tau)$ of the steady state bequest distribution depends on the bequest motive distribution and estate tax rate. As we do not impose any structural assumptions on the underlying bequest motive distribution, equation (6) provides the general formula for tax-revenue-maximizing tax rate under any exogenous bequest motive distribution. The first term in (6) gives the marginal revenue increase due to higher tax rate, while the second term measures the magnitude of efficiency loss from endogenous bequest tax base. Panel A of Table 1 delivers the revenue maximizing tax rate under different paramterization of CES utility function and different bequest motive distributions.

2.3 Steady state top welfare maximization

Maximizing tax revenue is certainly not the only fiscal objective of the government. Now we derive the optimal tax rate to maximize the social welfare. For the normative discussion, we could assign very general welfare weights to the households. Here we analyze the simplest case which transparently shows the equity-efficiency tradoff. We follows Saez (2001)'s setup and demonstrate the benchmark case where top welfare is utilitarian and tax revenue brings constant marginal social welfare. Consider a reference household with wealth A^* and bequest b^*, and identify the top wealth housholds as those who have wealth and bequest more than the reference household. From Theorem 2 we know that when A^* and b^* are sufficiently large, the stationary wealth and inheritance distribution will be asymptotic Pareto. We normalize without loss of generality the population of rich households to one. The government maximizes following social welfare function:

$$SWF = \max_{\tau} \int_{A^*}^{\infty} V_i di + p\tau R \int_{b^*}^{\infty} b_i di \tag{7}$$

where V_i represents the indirect utility of household i, and p measures the marginal social welfare of public provision. Following Saez (2001), we use p to transform top tax revenue linearly into social welfare. Again we discuss social welfare under stationary distribution. With concave individual utility, lower wealth inequality has implicit equity gains in the social welfare function. Equation (7) demonstrates in the simple way the efficiency-equity tradeoff concerning estate taxation: higher inheritance tax lowers inequality and has welfare implication (the first term in (7)); and inheritance tax determines total tax revenue and affects the redistributive impact of public services in social welfare (the second term in (7)).

Optimal household decision implies $V_i = C_i (A_i + \theta)^{1-\gamma}$ where C_i is a multiplier binding household's indirect utility with wealth. Since top wealth distribution has the same asymptotic Pareto tail as top bequest distribution[2], and individual marginal social welfare of wealth C_i is independent of wealth level, we can rewrite the aggregate top welfare as

$$\int_{A^*}^{\infty} V_i di = CA^{*1-\gamma} \frac{\lambda(\tau)}{\lambda(\tau) + \gamma - 1}$$

where $C \equiv E(C_i)$ is the average marginal social welfare of wealth. The above equation decomposes the effect of estate taxation on the welfare of top households into two channels: $CA^{*1-\gamma}$ is the indirect utility that embodies the effect how estate tax relates to each household; while $\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}$ uncovers how the top wealth distribution responds to estate tax.

Individual effect

Given fixed household wealth, higher estate tax τ induces lower proportional bequest and encourages household consumption. Intuitively, if estate tax is higher, of every dollar of bequest left for the next generation, a smaller fraction of it contributes to the household's utility. Equivalently, estate tax rate discounts the "effective" bequest motive $\chi_i (1-\tau)^{\alpha}$, making bequest less desirable for rich household. As a result, higher estate tax induces a lower bequest-consumption ratio, and makes consumption relatively more important in household utility. This effect shows the efficiency loss of estate taxation due to behavioral response from individual households. Since the marginal social welfare of wealth $\frac{C_i^{1-\gamma}}{1-\gamma}$ is decreasing with respect to tax rate τ, aggregated marginal social welfare C is lower under higher estate tax, representing a negative *individual effect.*

Distributional effect

From Theorem 2 we know asymptotic tail index $\lambda(\tau)$ is increasing in τ, which implies that higher estate tax induces lower top wealth inequality, and reduces wealth concentration within the top wealth community. If $0 < \gamma < 1$, aggregate multiplier C is positive, and large $\lambda(\tau)$ leads to smaller $\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}$ and worsen welfare. If $\gamma > 1$, C is negative, large $\lambda(\tau)$ also leads to larger $\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}$ and again worsen welfare. Thus estate

[2] If $\{A_i\}$ is asymptotically Pareto distributed, $\left\{A_i^{1-\gamma}\right\}$ is also asymptotically Pareo distributed.

taxation always entails negative *distributional effect*. This effect is similar to the standard efficiency distortion induced by taxation: as higher tax rate trims down the wealth concentration among the top households, smaller number of households are subject to the estate taxation and eligible to be accounted for in the social welfare function. Thus just as higher tax lowers the tax base, higher tax also shrinks the pool of top wealth households.

The decreasing utility from rich households is counterbalanced by the social benefits of redistributive tax revenue. Rewrite the total social welfare of public provision ase

$$p\tau R\int_{b^*}^{\infty} b_i = pRb^*\frac{\lambda(\tau)\,\tau}{\lambda(\tau)-1}$$

As in Saez (2001), p is the marginal social value of public provision, and it measures how government tranforms tax revenue into utility-comparable terms. Alternatively, $\frac{1}{p}$ measures the Pareto weight the "millionair" community is endowed with in social welfare function, and it reflects how much government cares about the welfare of the richest[3]. If $\frac{1}{p}=0$, welfare of the richest is silenced in social welfare calculation, and social welfare maximization collapes to tax revenue maximization. If $\frac{1}{p}$ is sufficiently large, the utility of the rich households predominates in the social welfare, and optimal estate tax would converge to zero. Since tail index $\lambda(\tau)$ is increasing with tax rate τ, estate tax induces a negative *distributional effect* from the tax base $\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}$, which represents the standard efficiency distortion generated by the shrinking tax base.

Social welfare maximization is equivalent to

$$\max_{\tau}\bar{g}\frac{C(\tau)\,\lambda(\tau)}{\lambda(\tau)+\gamma-1}+\frac{\tau\lambda(\tau)}{\lambda(\tau)-1} \tag{8}$$

Denote $\bar{g}=\frac{A^{*1-\gamma}}{pRb^*}$ as the ratio of marginal social value over the marginal utility. $\bar{g}$ measures how government weights the marginal utility of rich household against the marginal social value of public provision. $\bar{g}$ reveals how government evaluates equity-efficiency tradeoff, and the social welfare maximizing estate tax crucially hinges on this value. In Table 1 we discuss the effect of different values of $\bar{g}$ on social welfare maximizing estate tax. The maximization of equation (8) implies the following first order condition:

$$\bar{g}\left[C'(\tau)\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}-C(\tau)\frac{\lambda'(\tau)(1-\gamma)}{(\lambda(\tau)+\gamma-1)^2}\right]+\left[\frac{\lambda(\tau)}{\lambda(\tau)-1}-\tau\frac{\lambda'(\tau)}{(\lambda(\tau)-1)^2}\right]=0$$

The first term in the first square bracket gives the negative individual effect of inheritance tax on household utility, while the second term measures the

[3] Piketty and Saez (2013)'s positive optimal estate tax result hinges on the assumption that the rich household is endowed with relatively low Pareto weight.

2.4 Numerical results and discussions

We have the following degrees of freedom in our simulation: 1) exogenous bequest motive distribution $\{\chi_{ti}\}$; 2) exogenous labor productivity distribution $\{w_{ti}\}$; 3) CES utility coefficient α; 4) gross capital return R; 5) the social value of public provision $\bar{g}$. With these assumptions, the endogenous inheritance distribution will converge to asymptotic Pareto distribution. On this stationary distribution, we have the following calibration targets.

Interest rates

In the benchmark we assume an average working life of 50 years and an effective annual interest rate of 5%, this leads to a gross capital return R of 11.

Bequest motive distribution and bequest propensity

According to Lutz hendricks (2001) "Bequests and retirement wealth in the United States", "*70% of households receive no bequests. The children of the richest 2% of estates receive only 40% of parental terminal wealth. For the children of the very rich with estates greater than $20 million, the fraction inherited by children is only 15%*". According to Kopczuk and Lupton (2007) "To leave or not to leave: the distribution of bequest motives", from panel data of the Asset and Health Dynamics Among the Oldest Old (AHEAD) survey, "*roughly 3/4 of the elderly single population has a bequest motive, both the presence and the magnitude of the bequest motive are statistically and economically significant*" and "*among the elderly single households in the sample, about 4/5 of their net wealth will be bequeathed and approximately half of this is due to a bequest motive*". In our model, household bequest propensity depends on the CES utility parameter α, threshold level θ and the underlying bequest motive distribution. We calibrate these measures such that most of the households in the economy leave zero bequests. We consider the case where the bequest propensity is inverse Pareto distributed. If the bequest motive is sufficiently strong, the corresponding bequest propensity can be as high as 100%.

Wealth distribution

The wealth distribution in our model has the same asymptotic top inequality as the inheritance distribution, thus we use empirically estimated wealth inequality measure to calibrate the tail index of the endogenous inheritance distribution. Moreover, we make sure that when the estate tax rate ranges from 0% to 80%, the corresponding tail index of the inheritance distribution is within the range of 1 to 2.

In the simulation, we assume the utility function is Cobb-Douglas, and bequest propensity is inverse Pareto distributed. More specifically, we denote bequest propensity

$B = \frac{1}{x}$ and $x \sim Pareto\,(1, a)$ where a is the tail index of Pareto distribution. We assume the gross capital return $R = 11$, which corresponds to an annual capital return of 5% for 50 years. We find the feasible tail index a such that for a range of estate tax rates, the endogenous top wealth tail index is between 1 and 2.

Table 0

Panel A: Tail index of top wealth distribution for gross $R = 11$

a	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$	$\tau = 0.5$
0.07	1.213	1.296	1.400	1.534	1.717	1.981
0.08	1.134	1.214	1.314	1.443	1.620	1.875
0.09	1.064	1.141	1.237	1.362	1.532	1.779
0.10	1.000	1.074	1.168	1.289	1.453	1.693

Panel B: Tail index of top wealth distribution for $a = 0.10$

R	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$	$\tau = 0.5$
9.5	1.106	1.192	1.302	1.445	1.641	1.933
10	1.067	1.149	1.252	1.387	1.572	1.843
10.5	1.032	1.110	1.208	1.335	1.509	1.764
11	1.000	1.074	1.168	1.289	1.453	1.693

In Table 0 we can see that higher estate tax rate leads to larger tail index and lower top wealth inequality. Moreover, we analyze the effect of bequest propensity distribution and gross capital return on the tail index of endogenous wealth distribution. In Panel A, we see that larger a implies higher top wealth inequality. Since larger a means on average the society leads larger proportion of wealth as bequest, more altruistic society will generate stronger dynastic link across generations, and endure higher wealth inequality at the top. In Panel B, we confirm that higher gross capital return leads to higher top wealth inequality.

If $a = 0.1$, 5% of households will leave more than 60% of their lifetime wealth as bequest, while 75% of households will leave less than 5%. This distributional pattern generally matches that documented by Lutz Hendricks (2001).

In Table 1 we simulate the optimal estate tax rates under different social welfare functions and different bequest propensity distributions.

Table 1

Optimal estate tax τ

	gross return R =10			gross return R =10.5			gross return R =10.8		
	ratio of marginal social value over the marginal utility $\bar{g}$								
	$\bar{g}$ =0	$\bar{g}$ =0.1	$\bar{g}$ =0.2	$\bar{g}$ =0	$\bar{g}$ =0.1	$\bar{g}$ =0.2	$\bar{g}$ =0	$\bar{g}$ =0.1	$\bar{g}$ =0.2
Panel A: Concavity of social welfare function $\gamma = 0.1$									
shape parameter a									
0.09	50	41	28	47	34	23	44	28	17
0.10	43	27	15	31	14	9	22	10	5
Panel B: Concavity of social welfare function $\gamma = 0.5$									
shape parameter a									
0.09	50	46	42	47	42	38	44	37	33
0.10	43	35	30	31	27	23	22	18	16
Panel C: Concavity of social welfare function $\gamma = 0.9$									
shape parameter a									
0.09	50	48	45	47	45	40	44	39	37
0.10	43	37	36	31	30	27	22	18	18

Role of a

Larger a produces lower optimal estate tax rate. Larger a lifts the level of average bequest propensity $E(B) = \frac{a}{a+1}$ as more altruistic households leave larger proportion of wealth as beqeust. Since bequest propensity measures the correlation between bequest received and bequest left, larger B strengthens the dynamic linkage of generational bequest accumulation, thus raising top inequality and intensifying the wealth concentration at the top. On the one hand, from the formula of indirect utility multiplier C_i, larger B_i induces larger elasticity of household utility to estate tax, as $\frac{dC_i}{1-\tau}\frac{1-\tau}{C_i} = B_i$. As a result, if a amplifies the negative welfare feedback from the *individual effect*. On the other hand, lower top wealth inequality, or equivalently larger $\lambda(\tau)$, induces negative *distributional effect* from both welfare base $\frac{\lambda(\tau)}{\lambda(\tau)+\gamma-1}$ and tax base $\frac{\lambda(\tau)}{\lambda(\tau)-1}$[4]. From Lemma 3 we know that larger a further aggravates this negative effect. As a result, larger a intensifies the negative tradeoff from higher estate tax and exert downward pressure on optimal estate tax rate.

Role of R

Larger R produces lower optimal estate tax rate. Higher gross capital return increases the top wealth inequality. The effect on optimal tax rate is similar to that of a, except that R does not act directly on household's utility through the *individual effect*.

[4]If $0 < \gamma < 1$, aggregate multiplier C is positive, and large $\lambda(\tau)$ leads to smaller $\frac{C(\tau)\lambda(\tau)}{\lambda(\tau)+\gamma-1}$ and $\frac{\lambda(\tau)}{\lambda(\tau)-1}$. If $\gamma > 1$, C is negative, large $\lambda(\tau)$ also leads to smaller $\frac{C(\tau)\lambda(\tau)}{\lambda(\tau)+\gamma-1}$. Thus the distributional effect of estate taxation is always negative.

Role of γ

Except for the case where $\bar{g} = 0$, larger γ produces higher optimal estate tax rate. γ is the concavity of social welfare function and measures how much government values egalitarian distribution of wealth among top wealth households. The larger γ, the more important is taxation and redistribution in social welfare consideration, and the higher is the optimal estate tax.

Table 2

Optimal estate tax under different $\bar{g}$

$\bar{g}$	0	0.5	1	2	5	10	20
optimal τ	50	36	30	20	8	1	0

In Table 2, we focus on the case where bequest propensity distribution has shape parameter $a = 0.09$, gross capital return $R = 10$, social welfare concavity $\gamma = 0.9$. we simulate the optimal estate tax rates under different ratio of marginal social value over the marginal utility $\bar{g}$.

Role of $\bar{g}$

Larger $\bar{g}$ produces lower optimal estate tax rate. $\bar{g}$ represents the relative importance of top welfare against social value of public provision. Since higher estate tax deterioates the utility of rich households, larger $\bar{g}$ pivots goverments towards caring more about rich households, thus lowers the optimal estate tax rate. On the other hand, $\bar{g} = 0$ corresponds to the Rawsian welfare criterion and government only values redistribution. In this case, social welfare maximization collapes to tax revenue maximization. Thus social welfare maximizing estate tax rates are independent of concavity measure γ. Furthermore, in Table 2 we can see a clear monotone relationship between the $\bar{g}$ and optimal estate tax rate.

3 Inheritance taxation with elastic labor supply

Controversy on positive estate taxation has never been settled neither in academic arena nor in policy debate. Literature is scattered with different modelling of bequest motives and dynastic linkage. Cremer and Pestieau (2006) and Kopczuk (2013) provided a comprehensive survey on recent developments. In a very recent paper, Piketty and Saez (2013) pushes the policy debate on optimal estate taxation one step further by deriving an optimal tax formula that applies to a wide range of model settings. By using empirically estimable statistics, Piketty and Saez's formula gives a very clear decomposition of different forces underlying the efficiency-equity tradeoff and unifies many famous estate taxation results, nesting both Chamley-Judd's zero estate tax result and Farhi and Werning (2010)'s zero/negative estate tax result.

Piketty and Saez (2013)'s optimal estate tax formula is appealing in both positive and normative discussion. The formula merits from its close-form analyticality under very general utility function and general social welfare criterion. Their model features idiosyncratic utility and endogenized labor supply and bequest decisions. This setup is different from Farhi and Werning (2010) in two prominent ways. Firstly, in Farhi-Werning economy, parent receives no bequest and child leaves no bequest. Thus the economy has no inter-generational linkage nor aggregate dynamics. Households in Piketty-Saez economy both receive and leave bequest, and the generational linkage elicits rich dynastic bequest accumulation and long run aggregate capital convergence. The endogeneous stationary wealth distribution is what powers all the aggregate elasticity measures and distributional parameters[5]. The general idiosyncradic utility assumption is convenient to turn the dynamic wealth accumulation into a Markov process with stochastic multiplicative terms. Accordingly to Binhabib, Bisin and Zhu (2011) Theorem 1, under certain regulative conditions, it is sufficient to generate stationary wealth distribution and asymptotic Pareto Law.

Second and most crucially, generational linkage in Piketty-Saez economy brings about additional household heterogeneity in social welfare consideration. As wealth status is partially inherited by the next generation through bequests, stochastic labor earning fails to fully account for household wealth inequality. The additional dimension of wealth inequality greatly enriches the efficiency-equity tradeoff. As labor income cannot fully characterize wealth disparity, standalone labor income tax becomes insufficient as redistribution tool to smooth consumption across households. If political agenda pushes for redistribution and aims to alleviate poor-rich polarization, positive estate taxation becomes the natural next step. Furthermore, bequest received also creates addition arena for normative discussion as low bequest received could be the signal for need of government benefits.

However, with general utility function we cannot carry out comparative statistics and cannot distinguish the counteracting forces of taxation scheme on the underlying wealth distribution. In order to plug in the empirically obtained elasticities, Piketty and Saez (2013) focused on stationary distributions. However, since each tax scheme uniquely determines one stationary distribution, the empirical elasticities that pinned down the optimal tax rate should be generated exactly from the optimal distribution. The crucial assumption that carries through the argument is that the elasticities are the same across varying distributions. Instead of justifying this assumption, we relax it. To make our results comparable to Piketty and Saez (2013), we follow their household budget constraint and more importantly, the balanced government budget condition; but we specify the individual utility function to show explicitly the effect of taxation on underlying wealth accumulation and stationary wealth distribution.

[5] See definition of bequest elasticity e_B and $\hat{e}_B$, labor supply elasticity e_L, distributional parameters $\bar{b}^{received}$, $\bar{b}^{left}$ and $\bar{y}_L$ in Piketty and Saez (2013) equation 3 and 4.

Individual derives utility from consumption, leisure and bequest, and leisure $0 \leq 1 - l_{ti} \leq 1$. Household's utility $V\left(c_{ti}, 1 - l_{ti}, b_{t+1i}\right) = \frac{\left(u(c_{ti}, b_{t+1i})(1-l_{ti})^{\beta}\right)^{1-\gamma}}{1-\gamma}$, where $\gamma > 0$ and $\beta > 0$, and $u\left(c_{ti}, b_{t+1i}\right)$ is CES with coefficient $0 \leq \alpha < 1$. Households draw bequest motive and idiosyncradic labor income shocks and make optimal decision on resource allocation. We assume growth is exogenous and all varibles, including mean wage and bequest residual θ, are growing at constant rate g. We assume a constantly growing bequest threshold θ here to calibrate the bequest distribution observed in the data. After de-trended all the varibles, household utility maximization can be written as:

$$V\left(c_{ti}, l_{ti}, b_{t+1i}\right) = \frac{\left[g^{t}\left[c_{ti}^{\alpha} + \chi_{ti}\left(\left(1-\tau_{B}\right)\left(b_{t+1i}+\theta\right)\right)^{\alpha}\right]^{\frac{1}{\alpha}}\left(1-l_{ti}\right)^{\beta}\right]^{1-\gamma}}{1-\gamma}$$

such that

$$c_{ti} + g b_{t+1i} = b_{ti} R_{t}\left(1-\tau_{B}\right) + w_{ti} l_{ti}\left(1-\tau_{L}\right) + T_{t}$$

Households decisions are two-fold. First, similar to the last section, optimal household consumption-bequest decision implies

$$b_{t+1i} = \begin{cases} B_{ti}\left(A_{ti} - \hat{\theta}_{ti}\right) \text{ if } A_{ti} > \hat{\theta}_{ti} \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

where A_{ti} represents total household wealth. Bequest threshold $\hat{\theta}_{ti} = \left(\chi_{ti}/g\right)^{\frac{1}{\alpha-1}}\left(1-\tau_{B}\right)^{\frac{\alpha}{\alpha-1}}\theta$ and bequest propensity $B_{ti} = \frac{1}{g + (\chi_{ti}/g)^{\frac{1}{\alpha-1}}(1-\tau_{B})^{\frac{\alpha}{\alpha-1}}}$. Households under the threshold wealth level leave zero bequest, and households above it bequeath a share B_{ti} of lifetime wealth to the next generation.

Second, optimal labor-leisure decision implies following household labor supply function:

$$l_{ti} = \begin{cases} \frac{1}{1+\beta}\left[1 - \frac{b_{ti} R_{t}(1-\tau_{B}) + T_{t}}{\frac{1}{\beta} w_{ti}(1-\tau_{L})}\right] \text{ if } b_{ti} R_{t}\left(1-\tau_{B}\right) + T_{t} \leq \frac{1}{\beta} w_{ti}\left(1-\tau_{L}\right) \\ 0 \text{ otherwise} \end{cases} \tag{10}$$

Aside from the optimal bequest decisions, optimal labor supply is affected by both income and substitution effect of wage. If wage is high, substitution effect dominates the income effect, i.e. $b_{ti} R_{ti}\left(1-\tau_{B}\right) + T_{t} \leq \frac{1}{\beta} w_{ti}\left(1-\tau_{L}\right)$, household supplies positive amount of labor, and bequest accumulation is

$$b_{t+1i} = \frac{B_{ti} R_{t}\left(1-\tau_{B}\right)}{1+\beta} b_{ti} + B_{ti}\left[\frac{w_{ti}\left(1-\tau_{L}\right) + T_{t}}{1+\beta} - \hat{\theta}_{ti}\right] \tag{11a}$$

If wage is relatively low, income effect dominates the substitution effect, i.e. $b_{ti} R_{ti}\left(1-\tau_{B}\right) + T_{t} > \frac{1}{\beta} w_{ti}\left(1-\tau_{L}\right)$

$$b_{t+1i} = B_{ti} R_{t}\left(1-\tau_{B}\right) b_{ti} + B_{ti}\left[T_{t} - \hat{\theta}_{ti}\right] \tag{11b}$$

For the top wealth households, capitalized inheritance takes a dominant proportion in their lifetime wealth, thus they optimally choose to supply zero labor.

Bequest accumulation process converges to asymptotic Pareto distribution. The asymptotic Pareto index λ is defined by

$$E\left[BR\left(1-\tau_B\right)\right]^{\lambda}=1 \tag{12}$$

Higher estate taxation induces lower top wealth inequality as larger τ_B scales down multiplicative term $BR\left(1-\tau_B\right)$ in bequest accumulation process. We can see that redistribution of government tax revenue does not affect the wealth inequality at the top tail of wealth distribution. Intuitively, as we move towards the top tail of wealth distribution, accumulative effect of dynastic bequest takes the dominant role in wealth composition; and labor income and government transfer matters less for richer households.

The government collects taxes from total capitalized inheritance received and total labor income by current generation, then uses the tax revenue to finance the redistributive transfers. Government budget balance condition is represented by

$$T_t=R_t\tau_B\int_i b_{ti}+\tau_L\int_i w_{ti}l_{ti} \tag{13}$$

3.1 Steady state effect of estate taxation

to be continued

3.2 Numerical simulation: calibration

Let us conclude the optimal household decisions in the following steps: (1) household receives capitalized after-tax inheritance from the previous generation, receives government transfers and draws the labor productivity; (2) household supplies the optimal amount of labor according to equation (10); (3) with the optimal labor decision, household draws bequest motives, and make the optimal bequest leaving decision according to equation (9).

Each household receives inheritance b_i takes estate tax τ_B, labor tax τ_L and government transfers T as given, and draws bequest motive χ_i and labor productivity w_i, and makes optimal decisions of labor supply l_i and bequest left $b_i^{'}$. From every household, government collects inheritance tax $Rb_i\tau_B$ and labor tax $w_il_i\tau_L$. On the balanced growth path, total tax collected equals to total transfer expenditure. As a result, we have the following assumptions and calibration targets.

Assumption: labor productivity and bequest motive

Labor productivity distribution is assumed to be lognormal, i.e. $\ln l\sim N\left(\mu_l,\sigma_l\right)$. Bequest motive distribution is also assumed to be lognormal, i.e. $\ln\chi\sim N\left(\mu_\chi,\sigma_\chi\right)$.

Assumption: utility function

Utility function presents four parameters to be determined: CES coefficient α, labor-leisure substitution elasticity β, concavity coefficient γ, and bequest threshold θ. The first three parameters are obtained from empirical studies, while the last is calibrated such that the stationary bequest distribution features a large proportion of zero-bequest leavers.

Assumption: growth, capital return

Growth is exogenous and does not affect the dynamics in the bequest accumulation process. Thomas Piketty and his colleagues have estimated that the major western countries have gone through an annual growth of 2-3% and an annual capital return of 4-5% in the past several decades. Here we compound the annual rates to a 45 years working lifetime. Accordingly, we assume that gross growth rate $g = 2.5$ and gross capital return $R = 10$ in our benchmark simulation.

Calibration target: transfer

We fix a transfer target T, such that the required labor income tax will adjust one-on-one with varying inheritance tax to finance the budget. We also use T to calibrate the stationary wealth distribution such that total tax revenue to GDP ratio is around 24%, which is reported by OECD (2009) from U.S. data.

Calibration target: capital income to labor income ratio

We calibrate the model such that the aggregate capital income to labor income ratio is around 1:2, which is the literature standard.

Calibration target: wealth inequality

One crucial target of our simulation is to match the empirical wealth inequality. We calibrate the model such that the endogenous wealth inequality has empirical relevence, i.e. the tail index is between one and two.

3.3 Numerical simulation: results

to be continued

4 Conclusion

References

[1] Benhabib, J., Bisin, A. and Zhu, S. (2011), The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents. *Econometrica*, 79: 123-157.

[2] Kopczuk, W., and J. Lupton (2007), To Leave or Not to Leave: An Empirical Investigation of the Distribution of Bequest Motives," *Review of Economic Studies*, 74, 207-235.

[3] Piketty, T. and Saez, E. (2013), A Theory of Optimal Inheritance Taxation. *Econometrica*, 81: 1851–1886.

[4] Saez, E. (2001), Using Elasticities to Derive Optimal Income Tax Rates. *The Review of Economic Studies* 68: 205-229.

5 Appendix: