School enrollment, fertility differential, and development: theory and evidence

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Abstract

We present mechanics and observations of school enrollment and fertility differential in Brazil and Indonesia. Given the costs and levels of education, educated parents have a higher child school enrollment ratio but may have higher or lower fertility than do illiterate parents. Higher living costs or education subsidies increase school enrollment and reduce fertility for educated mothers first and for illiterate mothers later. This creates ebbs and flows for average output growth and alters fertility differential from the rich having higher fertility to the opposite. Census data estimations support a positive impact of parental education status on child school enrollment.

Keywords: School enrollment, Fertility differential, Growth, Demographic transition

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1 Introduction

Child school enrollment, fertility, and differentials in these variables across mothers with different education status have been recognized as important factors for human capital accumulation, economic growth, inequality, and the demographic transition. The conventional quantity-quality trade-off theory focuses on the choice of the same level of education for all siblings and postulates a higher opportunity cost of rearing a child for educated parents than that for illiterate parents. Thus, the conventional wisdom holds that illiterate parents choose more children and less child education than educated parents do, and that this fertility differential impedes economic growth and the demographic transition. However, there is evidence that educated parents have more children than illiterate parents in some stage of development or in some countries. Also, the existing theoretical analysis of child education has paid little attention to the fact that many parents in developing countries only enroll a fraction, rather than all, of their children in school. The underlying mechanism is poorly understood despite its important implications for growth, inequality arising within siblings, and priorities in public education policies.

In Brazil and Indonesia, for example, most mothers aged 40-49 in 1960 were illiterate and less than 10% of them completed primary education. Illiterate mothers in Brazil had over 2.5 times as large fertility as, but a 50 percentage points lower child school enrollment ratio than, what educated mothers had. Whereas illiterate mothers in Indonesia had 10% lower fertility and a 30 percentage points lower child school enrollment ratio, at odds with the standard negative relationship between fertility and maternal education. Since then, school enrollment ratios of children from both educated and illiterate mothers have increased substantially and remained higher for educated mothers, as portrayed in Figure 1 based on census data. Total fertility rates have declined substantially in both countries during 1960-2010, from 6.21 to 1.8 in Brazil and from 5.67 to 2.1 in Indonesia, as shown in Table 1, comparable to the demographic transition in the U.S. since 1800 in Greenwood and Seshadri (2002). In the meantime, youth illiteracy rates have also experienced a dramatic decline, from 46.8% to 0.9% in Brazil and from 42.6% to 5.6% in Indonesia.[1] During the same period 1960-2010, per capita GDP increased by a factor of 3 in Brazil and nearly 5 in Indonesia in Table 1. The 10-year average growth rates peaked first at 6% in both countries in 1970-1980 and again at 2% and 3.7% respectively in 2000-2010 in Brazil and Indonesia, following significant increases in school enrollment ratios in the preceding decades. Nevertheless, the fertility rate of better educated women in Indonesia declined faster than that of less educated women. In the later period, better educated Indonesian women indeed had a lower fertility rate than did illiterate women, suggesting an eventual transition from the Malthus to Becker regime once a country passed a certain development stage. In Vogl (2013) using microdata of 48 developing countries, the relationship between fertility and social economic status flipped from positive at the early stage of the demographic transition to negative, first in Latin America, then in Asia, and finally in Africa.

The purpose of the present paper is to build up a model of school enrollment ratios and fertility and use census data from Brazil and Indonesia to check school enrollment and fertility rates of different types of mothers by education status and over time. Our study sheds light on some problems and questions in development and government education policies. Do mothers of each type only enroll some rather than all children for education and hence choose diverse intergenerational mobilities within siblings? Do more educated mothers enroll a larger fraction of children in school? Does fertility differential among mothers with different education status flip directions in the development process? How do school enrollment and fertility differentials interact and affect economic growth and demographic transitions? Finally, can government policies increase school enrollment and influence economic growth and the demographic transition?

The causes and consequences of demographic changes have long been among the prime concerns of development. The early Malthus theory of population (e.g. Malthus, 1872) hypothesized a positive effect of income on fertility, exemplified by the argument

[1]The total fertility rate is extracted from the World Development Index. The youth illiteracy rate, extracted from Barro and Lee (2013), refers to the proportion of youth aged between 15 and 19 who do not have any formal schooling.

that poor reliefs to large families could lead to even more children from poor families and thus worsen poverty. Boyer (1989) indeed found empirical evidence of a positive fertility effect of the poor reliefs. However, the drastic fertility decline along with a steady rise in the standard of living in industrial nations eventually gave rise to the modern quantity-quality trade-off theory of fertility pioneered by Becker (1960). The demographic transition along with investment in children has been studied by Becker et al. (1990), Galor and Weil (2000), and Greenwood et al. (2005), among others.

Fertility differential, referring typically to the excess of children from the poor over that from the rich, has also been studied along with population growth and population quality for a long time. An inverse relationship between fertility and wealth appeared in British demographers' writings as early as 1660-1760 in the survey article of Kuczynski (1935) and received reconfirmation in many later studies in the review paper of Anastasi (1956). It caused concerns about an increasing proportion of children from poor families and a subsequent decline in the intellectual level of the population. These concerns are further reinforced by the observation that fertility declined at a faster rate in the upper than in the lower socioeconomic and education classes. Fortunately, such concerns have subsided as later studies showed that the average level of intelligence actually increased over time (e.g. Wheeler, 1942; Burt, 1950). Economists shared similar concerns about fertility differential in relation to the quantity and quality of children, income distribution, and economic growth; see, e.g., Kremer and Chen (2002), de la Croix and Doepke (2003) , Moav (2005), and Fan and Zhang (2013).[2] According to the literature, an economy with higher fertility differential tends to have higher income inequality, lower human capital investment, slower economic growth, and faster population growth. The underpinning includes the positive intergenerational correlation in income or education (e.g. Solon, 2002) and the trade-off between fertility and education. Comparing such conventional views with the contrasting observations in Brazil and Indonesia, it remains

[2] Lam (1986) pointed out that the relationship between fertility differential and inequality could be very sensitive to the choice of inequality measures as an increase in the fertility rate of poor families could increase the coefficient on variation but decrease the variance of log income. Consequently, the correlation between changes in the mean fertility rate and some aggregate measures of inequality may lead to misleading inferences.

puzzling why Brazil had a faster increase in child school enrollment and a steeper decline in average fertility than did Indonesia, despite starting with a much larger fertility differential. Moreover, fertility differential in these existing models does not flip signs when the economy develops.

The present paper develops a theory and presents empirical observations of fertility and child school enrollment of mothers with different education status. We first explore the underlying mechanics of the transition with illiterate and educated parents in Section 2 in an overlapping-generations economy with a wage premium for educated workers. In this model, parents choose the number of children and the fraction of children going to school, taking as given both the education cost, as in de la Croix and Doepke (2003), and the living cost per child, as in Becker et al. (1990). The remaining fraction of children who do not go to school can earn wage income, as in Doepke and Zilibotti (2005).

The assumption of parents choosing a fraction of children for education hinges on the fact that the education system is typically structured as fixed bundles, such as primary, secondary and tertiary education, other than continuously divisible subjects. This assumption has some advantages over the conventionally assumed parental choice of an equal level of education for all children, and leads to some new results. First, this assumption fits into data in which many parents enroll some rather than all of their children in school, and captures the downward and upward mobility not only across but also within families with respectively high or low social economic status. Second, it can create waves of high and low growth rates through increasing school enrollment ratios of mothers with different education status at different paces. Third, it helps to explain why fertility differential changes signs in the demographic transition.

In the present model, educated parents may have higher or lower fertility than do illiterate parents, depending on the relative strength of the income (Malthusian) and substitution (Beckerian) effects of parental wages. If the child living cost is sufficiently large (small) relative to the education cost and the child wage, then fertility is higher (lower) for educated parents than for illiterate parents and is increasing (decreasing) with parental wages. Intuitively, a higher child living cost strengthens the income effect of parental wages, but a higher child wage weakens this income effect. Whereas a higher education cost strengthens the substitution effect. Regardless of the sign of fertility differential, however, educated parents have higher child school enrollment ratios, due to a higher forgone wage for rearing a child, than do illiterate parents. Over time, the school enrollment ratio increases with the future wage premium to education, the forgone labor income rearing a child, the education subsidy, and the child living cost, but decreases with the education cost, the income tax rate, and the child wage. When wages and costs of children change in proportion, the school enrollment ratios remain constant on the balanced growth path. Education subsidization financed by income taxes is conducive to development, during which school enrollment rises and average fertility may initially increase but eventually fall. Surprisingly, a faster rise in the cost of living relative to wages also reduces fertility rates and increases school enrollment in contrast to a constant cost of living in Becker et al. (1990). The rise in school enrollment and the fall in fertility contribute to growth in output per worker in the future. Fertility differential flips directions in the early stage of development when educated mothers raise the fraction of children for education and reduce fertility ahead of illiterate mothers. Numerical simulations resemble observed increases in school enrollment ratios and declines in fertility, first for educated mothers and then for illiterate mothers. Thus, the simulation results show peaks and troughs in the growth rate and the switch of fertility differential as observed in data.

We describe the multiple-rounds of census data in Brazil and Indonesia from 1960 to 2010 and present empirical results in Section 3. Having multiple observations for each country enables us to remove the impact of country fixed-effects on the correlation between fertility differential and the other variables of interest. In contrast, with the exception of Vogl (2013), the previous empirical work on fertility differential, school enrollment, population growth, and economic growth is mostly based on cross sectional variations.[3] We use census data with a large sample size and a long sample period. Thus,

[3] Vogl (2013) used data from the Demographic and Health Survey (DHS). The advantage of using the DHS data is that it contains fertility information of 48 developing countries, whereas the disadvantage is its relatively small sample size and short sample period.

the current empirical investigation sheds some new lights on the puzzling evolution of the fertility distribution along with changes in education status across generations.

From the census data, the fertility rates in Brazil and Indonesia shared comparable downward trends although the fertility differential, measured by the ratio of the fertility rate of illiterate women to that of middle school graduates, was much larger in Brazil than in Indonesia. The average numbers of children born to women aged 40-49 were 5.51 and 5.27 for the 1920s birth cohort in Brazil and Indonesia, respectively, and 3.14 and 3.79 for the 1950s birth cohort. Accompanying the significant fertility decline was a dramatic reduction in women's illiteracy rates in both countries. The proportion of women with at least secondary school education actually increased faster in Brazil, which challenges the conventional wisdom that a higher fertility differential hinders human capital growth. The evidence confirms our model's prediction that a higher fertility differential does not necessarily slow down growth in human capital during the transition process. In both countries, logit regression results indicate stronger positive effects of higher maternal education status on children's school enrollment and overall positive trends of school enrollment in all maternal groups, all of which are statistically significant. This evidence supports our theoretical result that better educated mothers have higher child school enrollment ratios. Interestingly, the decline in mothers' illiteracy over time played an important role in reducing average fertility in Brazil with a high fertility differential but a much weaker role in Indonesia with a low fertility differential. In fact, the average fertility of the sampled mothers increased slightly in Indonesia during 1960-1980 along with falling illiteracy, as illiterate mothers had lower fertility than educated mothers did. Some concluding remarks will be given in Section 4.

2 The model and theoretical results

Consider an economy with discrete time, $t = 1, 2, \cdots$, and overlapping-generations of two period lived agents, children and parents. There are two types of parents: illiterate (or less educated) ones with human capital h_t^L, and educated ones with human capital

$h_t^H > h_t^L$. The production of a single final good uses effective labor $l_t^k h_t^k$ for $k = H, L$:

$$y_t^k = l_t^k h_t^k. \tag{1}$$

The wage rate of a parent is equal to her human capital $W_t^k = h_t^k$ and thus $W_t^H > W_t^L$.

2.1 *Education technologies*

The cost and level of education in the present model are externally structured to parents as opposed to the conventional assumption of divisible costs and divisible levels of education. The education cost per child, such as a tuition fee, is proportional to the human capital of educated adults, $e_t = \xi_t h_t^H$ where $\xi_t \in (0, 1)$. The human capital of a child through school education depends on

$$h_{t+1}^H = A(\psi + B\xi_t)^{\phi} h_t^H \tag{2}$$

and the human capital of a child without schooling depends on

$$h_{t+1}^L = A\psi^{\phi} h_t^H \tag{3}$$

where $A > 0$ is the efficiency parameter; $B > 0$ is the relative advantage of access to education for children enrolled in school; $\psi > 0$ captures the spillovers of the human capital of educated parents, h_t^H, to all children; and $0 < \phi < 1$ measures the degree of returns to the access to education.

From (2) and (3), the ratio of the high to low human capital of children, denoted as $\omega_{t+1} \equiv h_{t+1}^H / h_{t+1}^L$, is determined by

$$\omega_{t+1} \equiv \frac{h_{t+1}^H}{h_{t+1}^L} = \left(\frac{\psi + B\xi_t}{\psi} \right)^{\phi} > 1. \tag{4}$$

Since $0 < \phi < 1$, the return to education ω_{t+1} is increasing with the ratio of education spending to the human capital of educated adults, ξ_t, at a diminishing rate: $\omega'(\xi_t) > 0$

and $\omega''(\xi_t) < 0$. It is straightforward to extend the case for more types of individuals by education levels.

2.2 *Parental decisions on fertility and child school enrollment*

Each parent has one unit of time endowment and has to spend v units of time rearing a child. A parent chooses the number of children, $n_t^k \in [0, 1/v]$, and the fraction of them to go to school, $\lambda_t^k \in [0, 1]$, taking the cost and structure of education as given. The average human capital of children in the family, denoted as $\bar{h}_{t+1}^k$, is:

$$\bar{h}_{t+1}^k \equiv \lambda_t^k h_{t+1}^H + (1 - \lambda_t^k) h_{t+1}^L. \tag{5}$$

The preference of parents is a CRRA function of parental consumption c_t^k, the number of children n_t^k, and the average human capital of children $\bar{h}_{t+1}^k$ in the family:

$$U_t^k = \frac{\left(c_t^k\right)^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left[\left(n_t^k\right)^{\alpha} \left(\bar{h}_{t+1}^k\right)^{\eta}\right]^{1-\gamma} - 1}{1-\gamma}, \quad 1 > \alpha, \eta > 0, \quad \sigma > 0, \quad \gamma > 0. \tag{6}$$

A type-k parent has a wage rate W_t^k and an income tax rate $\tau_t^k \in [0, 1)$. Part of the income tax revenue finances an education subsidy at a rate $s_t^k \in [0, 1)$. A child without going to school can earn a wage W_t^C in an informal sector. Earnings from child labor will be added to parental budgets as in Doepke and Zilibotti (2005). In addition to the cost of enrolling a child to school (e_t) mentioned earlier, there is a living cost per child d_t. Both types of costs are exogenously given to families. The budget constraint of a parent is given by

$$c_t^k = (1 - \tau_t^k) W_t^k (1 - v n_t^k) + W_t^C (1 - \lambda_t^k) n_t^k - d_t n_t^k - (1 - s_t^k) e_t \lambda_t^k n_t^k. \tag{7}$$

From (1) and (7), we have $l_t^k = 1 - v n_t^k$ and $W_t^k = h_t^k$.

The budget feasible set is non-convex due to the product of choice variables $\lambda_t^k n_t^k$. To permit an optimal interior solution, we need a restriction $1 > \alpha > \eta > 0$ as in Ehrlich and Lui (1991).

Assumption 1. $1 > \alpha > \eta > 0$.

This restriction means that the taste for the number of children is stronger than the taste for the quality of children.

A parent maximizes utility in (6) by choosing the number of children $n_t^k \in [0, 1/v]$ and the school enrollment ratio $\lambda_t^k \in [0, 1]$ subject to (5) and (7). The choice of enrolling a fraction of children for education is more practical when parents cannot afford to send all children to school, given the costs and levels of education, than the conventional assumption of giving all children an equal level of education. This choice of partial school enrollment is also consistent with Parish and Willis (1993) and our own observations to be presented later. With this choice, inequality will arise from children not only across families as in the literature but also within the same family.

The first-order condition with respect to the number of children n_t^k is

$$\begin{aligned}\frac{\partial U_t^k}{\partial n_t^k} &= -(c_t^k)^{-\sigma}[(1-\tau_t^k)W_t^k v + d_t + (1-s_t^k)e_t\lambda_t^k - W_t^C(1-\lambda_t^k)] \\ &\quad +\alpha\beta(n_t^k)^{\alpha(1-\gamma)-1}(\bar{h}_{t+1}^k)^{\eta(1-\gamma)} \leq 0. \qquad (8)\end{aligned}$$

The first term on the right-hand side is the forgone marginal utility for an additional child through the time cost $(1-\tau_t^k)W_t^k v$, the living cost d_t, and the education cost $(1-s_t^k)e_t\lambda_t^k$, net of child labor income $W_t^C(1-\lambda_t^k)$. The second term is the marginal utility gained from having an additional child.

If the child labor income is no less than the sum of the time cost and the living cost per child, $W_t^C \geq (1-\tau_t^k)W_t^k v + d_t$, then the net marginal benefit of having a child $\partial U_t^k/\partial n_t^k$ will be positive, implying a corner solution in which fertility equals its maximum $n_t^k = 1/v$. Since $W_t^H > W_t^L$, it is possible that illiterate parents choose the maximum level of fertility, whereas educated parents choose a lower level of fertility.

The first-order condition with respect to the school enrollment ratio λ_t^k is

$$\begin{aligned}\frac{\partial U_t^k}{\partial \lambda_t^k} &= -(c_t^k)^{-\sigma}n_t^k[(1-s_t^k)e_t + W_t^C] \\ &\quad +\beta\eta(h_{t+1}^H - h_{t+1}^L)(n_t^k)^{\alpha(1-\gamma)}(\bar{h}_{t+1}^k)^{\eta(1-\gamma)-1} \leq 0. \qquad (9)\end{aligned}$$

The first term on the right-hand side is the marginal cost for sending children to school, including a direct education cost $(1-s_t^k)e_t n_t^k$ as well as the forgone child labor income $n_t^k W_t^C$. The second term is the marginal utility obtained from sending children to school, driven by an education return $h_{t+1}^H - h_{t+1}^L > 0$ in the future.

Let us define a vector $P_t^k \equiv (W_t^C, \omega_{t+1}, d_t, e_t, s_t^k, \tau_t^k)$ and a real-valued function below

$$\Lambda^k(W_t^k, P_t^k) \equiv \frac{\eta(\omega_{t+1}-1)[(1-\tau_t^k)W_t^k v + d_t - W_t^C] - \alpha[(1-s_t^k)e_t + W_t^C]}{(\alpha-\eta)(\omega_{t+1}-1)[(1-s_t^k)e_t + W_t^C]}. \tag{10}$$

Equations (8)-(10) lead to the solution for the school enrollment ratio:

$$\lambda_t^k = \begin{cases} \min\left\{1, \max\left\{0, \Lambda^k(W_t^k, P_t^k)\right\}\right\} & \text{if } (1-\tau_t^k)W_t^k v + d_t > W_t^C; \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Equation (10) captures how strongly a parent wants to enroll children for education, given the return to education, the cost of education, the wage of the parent, the child living cost, the child wage, the tax rate, and the subsidy rate. In (11), parents send children to school only when the forgone parental earnings plus the living cost per child is greater than child labor earnings, $(1-\tau_t^k)W_t^k v + d_t > W_t^C$. Otherwise, parents would not enroll any child to school, and, as noted above, would choose the maximum number of children $n_t^k = 1/v$. Also, the rate of return to education $\omega_{t+1} - 1 = (h_{t+1}^H - h_{t+1}^L)/h_{t+1}^L = (W_{t+1}^H - W_{t+1}^L)/W_{t+1}^L$ has to be large enough to motivate parents to enroll a child to school.

Intuitively, the child school enrollment ratio, once positive and less than one, is increasing with the return to education $\omega(\xi_t)$, the parental wage W_t^k, the education subsidy rate s_t^k, and the taste for the average future earnings of children η, but decreasing with the education cost $e_t = \xi_t h_t^H$, the income tax rate τ_t^k, the child labor earnings W_t^C, and the taste for the number of children α. Note that a higher ratio of education spending to the human capital of educated parents, ξ_t, has a positive effect on school enrollment through the return to education and a negative effect on school enrollment through the cost of education. Also, the child school enrollment ratio is increasing with the forgone earnings for rearing a child $(1-\tau_t^k)W_t^k v$ and the child living cost d_t because such costs tend to tip the tradeoff between the quality and quantity of children in favor of the former. If wages, the cost of living, and the cost of education grow at the same rate in (11), then the school enrollment ratio remains constant over time for constant tax/subsidy rates.

We summarize the results as:

Proposition 1. *If $(1-\tau_t^k)W_t^k v + d_t > W_t^C$ and $0 < \Lambda_t^k < 1$, then the school enrollment ratio of children from type-k parents increases with the return to education ω_{t+1}, the forgone labor income $W_t^k v$ for rearing a child, the education subsidy rate s_t^k, and the child living cost d_t, but decreases with the education cost e_t, the income tax rate τ_t^k, and the child wage W_t^C. If wages, the cost of living, and the cost of education grow at the same rate, then the school enrollment ratio is constant over time for constant tax/subsidy rates.*

Among these results, the positive impact of a higher living cost on the school enrollment ratio differs from the conventional view in the literature. For instance, in Becker et al. (1990), a constant living cost per child is essential to determine two stages of development: a Malthusian stage starting at sufficiently low human capital and a growth path at sufficiently high human capital relative to the living cost. The intuition for the positive effect of a higher living cost on school enrollment becomes more transparent after we look at how it affects fertility. Moreover, countries with higher income do not necessarily have higher school enrollment ratios, which helps to explain why education subsidy rates are usually higher in more developed countries with higher school enrollment ratios.

Unlike the school enrollment ratio, fertility has no explicit solution for $\sigma \neq 1$ or $\gamma \neq 1$. In a special case with log utility $\sigma = \gamma = 1$, the solution for fertility is

$$n_t^k = \begin{cases} \frac{1}{v} & \text{if } (1-\tau_t^k)W_t^k v + d_t \leq W_t^C, \\ \frac{\alpha\beta(1-\tau_t^k)W_t^k}{(1+\alpha\beta)[(1-\tau_t^k)W_t^k v + d_t + (1-s_t^k)e_t\lambda_t^k - W_t^C(1-\lambda_t^k)]} & \text{else if } \sigma = \gamma = 1. \end{cases} \tag{12}$$

In (12), the parental wage, W_t^k, affects fertility in opposite directions. On the one hand, there is a positive effect through the potential parental labor income in the numerator (the Malthusian effect). On the other hand, there is a negative effect through the forgone labor income for rearing a child in the denominator (the Beckerian effect), directly and indirectly via the school enrollment ratio for $0 < \lambda_t^k < 1$. To determine the net effect of parental wage and other factors on fertility, let us derive the reduced-form solution for fertility from (7), (10), (11) and (12) for log utility $\sigma = \gamma = 1$:

$$n_t^k = \begin{cases} \frac{1}{v} & \text{if } (1-\tau_t^k)W_t^k v + d_t \leq W_t^C; \\ & \text{else if } \sigma = \gamma = 1: \\ \frac{\alpha\beta(1-\tau_t^k)W_t^k}{(1+\alpha\beta)[(1-\tau_t^k)W_t^k v + d_t - W_t^C]}, & \forall\ \Lambda_t^k \leq 0; \\ \frac{(\alpha-\eta)\beta(1-\tau_t^k)W_t^k(\omega_{t+1}-1)}{(1+\alpha\beta)\{[(1-\tau_t^k)W_t^k v + d_t - W_t^C](\omega_{t+1}-1) - (1-s_t^k)e_t - W_t^C\}}, & \forall \Lambda_t^k \in (0,1); \\ \frac{\alpha\beta(1-\tau_t^k)W_t^k}{(1+\alpha\beta)[(1-\tau_t^k)W_t^k v + d_t + (1-s_t^k)e_t]}, & \forall\ \Lambda_t^k \geq 1. \end{cases} \tag{13}$$

Fertility depends on wages and costs in various ways:

Proposition 2. *Suppose that* $(1 - \tau_t^k)W_t^k v + d_t > W_t^C$ *and that* $\sigma = \gamma = 1$. *The effect of raising parental wage* W_t^k *on fertility is*

(i) $\gtreqless 0$ *if* $\Lambda_t^k \leq 0$ *and* $d_t \gtreqless W_t^C$*;*

(ii) $\gtreqless 0$ *if* $0 < \Lambda_t^k < 1$ *and* $(\omega_{t+1} - 1)(d_t - W_t^C) \gtreqless (1 - s_t^k)e_t + W_t^C$*;*

(iii) > 0 *if* $\Lambda_t^k \geq 1$.

Also, fertility decreases with the child living cost d_t *in cases (i)-(iii), with the return to education* ω_{t+1} *in case (ii), and with the education cost* $(1 - s_t^k)e_t$ *in case (iii), but increases with the child wage* W_t^C *in cases (i) and (ii) and with the education cost in case (ii). If wages, the cost of living, and the cost of education grow at the same rate, then fertility is constant over time for constant tax/subsidy rates.*

Proof. The results in cases (i) and (iii) are transparent. The effects of d_t, e_t, W_t^C, and

ω_{t+1} are also transparent. The effect of W_t^k on fertility in case (ii) is given as

$$\frac{\partial n_t^k}{\partial W_t^k} = \frac{(\alpha-\eta)\beta(\omega_{t+1}-1)(1-\tau_t^k)[(d_t-W_t^C)(\omega_{t+1}-1)-(1-s_t^k)e_t-W_t^C]}{(1+\alpha\beta)\{[(1-\tau_t^k)W_t^k v+d_t-W_t^C](\omega_{t+1}-1)-(1-s_t^k)e_t-W_t^C\}^2}$$

$$\gtreqqless 0, \text{ if } (1-\tau_t^k)W_t^k v + d_t \gtreqqless W_t^C,$$

because $\alpha > \eta$ in Assumption 1 and $\omega > 1$. If wages, the cost of living, and the cost of education grow at the same rate, then fertility is constant over time for constant tax/subsidy rates according to (13). *Q.E.D.*

The conditions signing the effects of parental wages on fertility in Proposition 2 are novel. In one extreme case when the school enrollment ratio is equal to zero ($\Lambda^k \leq 0$), fertility increases (decreases) with the parental wage if the child living cost is greater (smaller) than the child wage as the income effect of the parental wage dominates (is dominated by) the substitution effect of the parental wage. Intuitively, a higher child living cost strengthens the income effect of the parental wage, whereas a higher child wage weakens the income effect of the parental wage. When these two factors offset $d_t = W_t^C$, log utility would imply that the income and substitution effects of parental wages should cancel out exactly in case (i). In the other extreme case when the school enrollment ratio is equal to 100% ($\Lambda^k \geq 1$), children do not work and thus fertility is increasing with the parental wage in case (iii) since now the income effect dominates.

In between, the effect of parental wages on fertility is complicated by the tradeoff between the quality and quantity of children ($0 < \Lambda^k < 1$) in case (ii): It is positive (negative) if the child living cost relative to the education cost and the child wage is so high (low) that $d_t > (<)[(1-s_t^k)e_t + W_t^C\omega_{t+1}]/(\omega_{t+1}-1)$. Moreover, a higher cost of education tips the quality-quantity tradeoff in favor of the latter and, conversely, a higher return to education tips the tradeoff against the latter. It is worth noting that the effect of the education cost on fertility has opposite signs in cases (ii) and (iii). In case (iii) when all children go to school, a higher education cost reduces fertility. In all cases, however, fertility is deceasing with the living cost per child, which helps to explain why school enrollment is increasing with the living cost per child as in Proposition 1. If wages, the cost of living, and the cost of education grow at the same rate, then fertility is constant over time for constant tax/subsidy rates and for logarithmic utility that allows for the full cancellation of the proportionate income and substitution effects.

Because of the wage differential, parents with different education status may belong to different cases (i), (ii) or (iii). Thus, they may respond to changes in wages and other factors differently when they choose the school enrollment and the number of children. We now explore the implications of (11) and (13) for differentials in child school enrollment and in fertility in turn.

From (11), particularly important for school enrollment differential is the forgone wage income for rearing a child $W_t^k v$: Educated parents should have higher child school enrollment ratios than do illiterate parents because $v(W_t^H - W_t^L) > 0$ when taxes and education subsidies are equal for all parents.

Proposition 3. *Suppose that the rates of taxes and subsidies are equal across agents, and that* $(1-\tau_t)W_t^k v + d_t > W_t^C$. *For* $0 < \Lambda_t^H \le 1$, *educated parents have higher child school enrollment ratios than do illiterate parents; otherwise, either no child goes to school for* $\Lambda_t^H \le 0$, *or every child goes to school for* $\Lambda_t^L \ge 1$.

Proof. Let $\tau_t^k = \tau_t$ and $s_t^k = s_t$ be equal rates of taxes and subsidies. Equation (10) implies

$$\Lambda_t^H - \Lambda_t^L = \frac{\eta(1-\tau_t)v(W_t^H - W_t^L)}{(\alpha-\eta)[(1-s_t)e_t + W_t^C]} > 0$$

under Assumption 1. From this and (11), for $0 < \Lambda_t^H \le 1$, $\lambda_t^H - \lambda_t^L > 0$. For $\Lambda_t^H \le 0$, $\Lambda_t^L < \Lambda_t^H \le 0$ and thus $\lambda_t^H = \lambda_t^L = 0$. For $\Lambda_t^L \ge 1$, $\Lambda_t^H > \Lambda_t^L \ge 1$ and thus $\lambda_t^H = \lambda_t^L = 1$. *Q.E.D.*

The positive relationship between the school enrollment ratio of children and maternal education status is consistent with the overwhelming evidence; see Haveman and Wolfe (1995) for a review on this issue based on the US data. As to be shown later, this positive correlation is also observed in both Brazil and Indonesia.

Proposition 4. *Suppose that the rates of taxes and subsidies are equal across agents, and that* $(1-\tau_t)W_t^k + d_t > W_t^C$. *With logarithmic utility,* $n^H \gtreqless n^L$ *if either of (i)-(iv) applies:*

(i) $\Lambda_t^H \leq 0$ *and* $d_t \gtreqless W_t^C$*;*

(ii) $\Lambda_t^L \leq 0 < \Lambda_t^H < 1$ *and*
$(\omega_{t+1}-1)(d_t - W_t^C) \gtreqless (1-s_t)e_t + W_t^C + \frac{W_t^H \Theta_t^L}{\alpha(W_t^H - W_t^L)}$ *where*
$\Theta_t^L \equiv \eta(\omega_{t+1}-1)[(1-\tau_t)W_t^L v + d_t - W_t^C] - \alpha[(1-s_t)e_t + W_t^C] \leq 0$*;*

(iii) $0 < \Lambda_t^L < \Lambda_t^H < 1$ *and* $(\omega_{t+1}-1)(d_t - W_t^C) \gtreqless (1-s_t)e_t + W_t^C$*;*

(iv) $0 < \Lambda_t^L < 1 \leq \Lambda_t^H$ *and*
$(\omega_{t+1}-1)(d_t - W_t^C) \gtreqless (1-s_t)e_t + W_t^C + W_t^L \frac{\{(\alpha-\eta)(\omega_{t+1}-1)[(1-s_t)e_t + W_t^C] - \Theta_t^H\}}{\alpha(W_t^H - W_t^L)}$
where $\Theta_t^H \equiv \eta(\omega_{t+1}-1)[(1-\tau_t)W_t^H v + d_t - W_t^C] - \alpha[(1-s_t)e_t + W_t^C] > 0$
and $\{(\alpha-\eta)(\omega_{t+1}-1)[(1-s_t)e_t + W_t^C] - \Theta_t^H\} \leq 0$.

If $\Lambda_t^L \geq 1$*, then educated parents have higher fertility than do illiterate parents.*

Proof. Let $\tau_t^k = \tau_t$ and $s_t^k = s_t$ be equal rates of taxes and subsidies. For $\sigma = \gamma = 1$, denote the denominator of fertility n_t^k in (13) as D_t^k, which is greater than 0 for $W_t^L v + d_t > W_t^C$. The fertility differential differs across these cases according to (13).

In case (i), the fertility differential is determined by

$$n_t^H - n_t^L = \frac{\alpha\beta(1-\tau_t)(W_t^H - W_t^L)(d_t - W_t^C)}{(1+\alpha\beta)D_t^H D_t^L},$$

which is signed by $d_t - W_t^C$, implying that educated mothers have more children than do illiterate mothers if the child living cost is higher than the child wage.

In case (ii), $n_t^H - n_t^L$ can be expressed as

$$n_t^H - n_t^L = \beta(1-\tau_t)\frac{\{\alpha(W_t^H - W_t^L)[(\omega_{t+1}-1)(d_t - W_t^C) - (1-s_t)e_t - W_t^C] - W_t^H \Theta_t^L\}}{(1+\alpha\beta)D_t^H D_t^L},$$

which is signed by the numerator. Because $\Lambda_t^L < 0$ implies $\Theta_t^L < 0$ (the numerator of Λ_t^L), educated mothers have more children than do illiterate mothers as long as $(\omega_{t+1} -$

$1)(d_t - W_t^C) \geq (1 - s_t)e_t + W_t^C$. This condition will be true when the living cost is sufficiently high and/or the education cost is sufficiently low. However, the relationship between n^H and n^L is ambiguous if $(\omega_{t+1} - 1)(d_t - W_t^C) < (1 - s_t)e_t + W_t^C$. In the case where education is very costly compared with the living cost, educated mothers may have fewer children than do illiterate mothers.

In case (iii), $n_t^H - n_t^L$ is determined by

$$\begin{aligned} n_t^H - n_t^L &= \frac{(\alpha-\eta)\beta(\omega_{t+1}-1)(1-\tau_t)(W_t^H - W_t^L)}{(1+\alpha\beta)D_t^H D_t^L} \\ &\times \quad [(\omega_{t+1}-1)(d_t - W_t^C) - (1-s_t)e_t - W_t^C] \end{aligned}$$

which is signed by $[(\omega_{t+1} - 1)(d_t - W_t^C) - (1 - s_t)e_t - W_t^C]$. Now, educated parents have more (fewer) children than illiterate parents do when the living cost is sufficiently high (low) relative to the education cost and child wage so that $(\omega_{t+1} - 1)(d_t - W_t^C)$ is greater (smaller) than $(1 - s_t)e_t + W_t^C$.

In case (iv), $n_t^H - n_t^L$ is determined by

$$\begin{aligned} n_t^H - n_t^L &= \frac{\beta(1-\tau_t)}{(1+\alpha\beta)D_t^H D_t^L} \{\alpha(W_t^H - W_t^L)[(\omega_{t+1}-1)(d_t - W_t^C) - (1-s_t)e_t - W_t^C] \\ &- \quad W_t^L[(\alpha-\eta)(\omega_{t+1}-1)[(1-s_t)e_t + W_t^C] - \Theta_t^H]\} \end{aligned}$$

which is signed by the value of the braces. Since $[(\alpha-\eta)(\omega_{t+1}-1)[(1-s_t)e_t+W_t^C]-\Theta_t^H]$ is the difference between the denominator and the numerator of Λ_t^H in equation (10), it is negative if $\Lambda_t^H \geq 1$. Therefore, educated mothers have more children than illiterate mothers do if $(\omega_{t+1} - 1)(d_t - W_t^C) \geq (1 - s_t)e_t + W_t^C$ as in case (iii) or even if this condition is relaxed to the extent such that the value of the braces is positive.

In the final case with $\Lambda_t^L \geq 1$, $n_t^H - n_t^L$ is determined by

$$n_t^H - n_t^L = \frac{\alpha\beta(W_t^H - W_t^L)(1-\tau_t)[d_t + (1-s_t)e_t]}{(1+\alpha\beta)D_t^H D_t^L} > 0.$$

Q.E.D.

The mixed results for fertility differential in cases (i) to (iv) of Proposition 4 are also in line with the mixed observations of fertility differentials in Brazil and Indonesia as aforementioned and to be detailed later. At the beginning when wages are too low even for educated parents to enroll children in school, as in case (i), educated parents have more (fewer) children than do illiterate parents if the child living cost is higher (lower) than the child wage. The difference between the child living cost and the child wage strengthens the income effect of the parental wage on fertility in the same way as mentioned earlier. When the child living cost is just equal to the child wage, then fertility is the same for all parents $n_t^k = \alpha\beta/[v(1+\alpha\beta)]$ from (13), because the income and substitution effects are now fully canceled out with log utility.

In case (ii), wages are high enough for educated parents to send just a fraction of children to school but are still too low for illiterate parents to send any children for education. In this case, the condition for the fertility rate of educated parents to be higher (lower) than that of illiterate parents is less (more) restrictive than the condition for fertility to increase (decrease) with the parental wage in case (ii) of Proposition 2. This is because now the gap in parental wages consists of two parts: first from the lower wage of illiterate parents to the critical level for child education, and then to the higher wage of educated parents. If the condition for the rise in the parental wage in the second part to increase fertility in (ii) of Proposition 2 is satisfied $(\omega_{t+1}-1)(d_t-W_t^C) > (1-s_t)e_t+W_t^C$, then the condition for the rise in the parental wage in the first part to increase fertility in (i) of Proposition 2 must be satisfied $d_t - W_t^C > 0$ as well.

In case (iii), wages are high enough for all types of parents to send a fraction, not all, of their children to school. Now, the condition for educated parents to have more (fewer) children than do illiterate parents is exactly the same as that for a rise in the parental wage to increase fertility in Proposition 2 (ii). In case (iv) with even higher wages, educated parents send all children to school, while illiterate parents send a faction of children to school. Now, the condition for educated parents to have higher fertility than do illiterate parents is less restrictive than the condition for fertility to increase with the parental wage in Proposition 2 (ii). This is because the increase in the wage of educated parents who send all children to school increases fertility as stated in Proposition 2 (iii).

The results in Propositions 1-4 help to explain the demographic transition, the change in school enrollment, as well as the evidence on the sign switch of fertility differential in Vogl (2013) across nations and in our own empirical findings in Section 3. Starting in early development when wages are low relative to the child living cost, illiterate parents have not only lower school enrollment but likely fewer children than do educated parents. Also, increases in parental wages during early development may induce higher fertility but little change in school enrollment, unless governments reduce the cost of education through subsidization. In a later stage of development when wages and education costs are high relative to the child living cost, however, fertility is lower for better educated parents, and further increases in parental wages lead parents to reduce fertility and increase child school enrollment ratios. In the end when all children finish all levels of education, the present model predicts that fertility will rise with wages again under the assumption of log utility. According to Myrskylä et al. (2009), developed countries with the human development index (HDI) value exceeding 0.9 have observed increases in fertility along with increases in HDI in the last decade. These countries have very high tertiary enrollment ratios, e.g. over 80% in Australia and over 90% in the US. Our results may help explain this observation. Next, we explore the equilibrium transition.

2.3 *Equilibrium transition*

To analyze the equilibrium transition, it is convenient to start with logarithmic utility. Let N_t^k be the number of type k ($k = H, L$) parents in period t. It is plausible to allow for higher tax rates for educated parents and higher subsidy rates for children of illiterate parents, i.e. $\tau_t^H \geq \tau_t^L$ and $s_t^H \leq s_t^L$. The government runs a balanced budget:

$$N_t^H(1 - vn_t^H)W_t^H\tau_t^H + N_t^L(1 - vn_t^L)W_t^L\tau_t^L = N_t^H\lambda_t^H e_t s_t^H + N_t^L\lambda_t^L e_t s_t^L + G_t, \qquad (14)$$

where G_t is government spending excluding education subsidies.

From $e_t = \xi_t W_t^H$, $W_t^k = h_t^k$, and a given state $(h_t^H, h_t^L, N_t^H, N_t^L, W_t^C)$, we substitute human capital levels into (10) and (13) for parental wages to obtain the equilibrium solution:

$$\Lambda_t^k = \frac{\eta(\omega(\xi_t) - 1)[(1 - \tau_t^k)h_t^k v + d_t - W_t^C] - \alpha[(1 - s_t^k)\xi_t h_t^H + W_t^C]}{(\alpha - \eta)(\omega(\xi_t) - 1)[(1 - s_t^k)\xi_t h_t^H + W_t^C]}, \tag{15}$$

$$n_t^k = \begin{cases} \frac{1}{v} & \text{if } (1-\tau_t^k)h_t^k v + d_t \leq W_t^C; \\ & \text{else if } \sigma = \gamma = 1: \\ \frac{\alpha\beta(1-\tau_t^k)h_t^k}{(1+\alpha\beta)[(1-\tau_t^k)h_t^k v + d_t - W_t^C]}, & \forall \Lambda_t^k \leq 0; \\ \frac{(\alpha-\eta)\beta(\omega(\xi_t)-1)(1-\tau_t^k)h_t^k}{(1+\alpha\beta)\{(\omega(\xi_t)-1)[(1-\tau_t^k)h_t^k v + d_t - W_t^C] - (1-s_t^k)\xi_t h_t^H - W_t^C\}}, & \forall \Lambda_t^k \in (0,1); \\ \frac{\alpha\beta(1-\tau_t^k)h_t^k}{(1+\alpha\beta)[(1-\tau_t^k)h_t^k v + d_t + (1-s_t^k)\xi_t h_t^H]}, & \forall \Lambda_t^k \geq 1. \end{cases} \tag{16}$$

The equilibrium school enrollment ratio λ_t^k follows (11) and (15). The equilibrium relationships between taxes and subsidies are given in (14).

The number of type-L individuals in generation $t+1$ can be written as

$$N_{t+1}^L = N_t^L n_t^L (1 - \lambda_t^L) + N_t^H n_t^H (1 - \lambda_t^H), \tag{17}$$

and the number of type-H individuals in generation $t+1$ as

$$N_{t+1}^H = N_t^L n_t^L \lambda_t^L + N_t^H n_t^H \lambda_t^H. \tag{18}$$

Let $\rho_t \equiv N_t^L / N_t^H$ be the ratio of illiterate to educated parents by number, and $\mu_t \equiv n_t^L / n_t^H$ be the fertility differential measure. Dividing both sides of (17) and (18) by the number of children of educated parents $N_t^H n_t^H$ gives

$$\frac{N_{t+1}^L}{N_t^H n_t^H} = \rho_t \mu_t (1 - \lambda_t^L) + (1 - \lambda_t^H), \tag{19}$$

$$\frac{N_{t+1}^H}{N_t^H n_t^H} = \rho_t \mu_t \lambda_t^L + \lambda_t^H. \tag{20}$$

Dividing (19) by (20) yields the transition equation of the ratio of illiterate to educated parents:

$$\rho_{t+1} = \frac{\rho_t \mu_t (1-\lambda_t^L) + (1-\lambda_t^H)}{\rho_t \mu_t \lambda_t^L + \lambda_t^H}. \tag{21}$$

From (21), the transition of relative education status across generations from a given level of ρ_t to ρ_{t+1} depends on school enrollment ratios and fertility differential. A higher child school enrollment ratio of all parents decreases the future portion of illiterate parents because $\partial\rho_{t+1}/\partial\lambda_t^L < 0$ and $\partial\rho_{t+1}/\partial\lambda_t^H < 0$. However, a rise in fertility differential increases the future ratio of illiterate to educated parents under Proposition 3, because

$$\frac{\partial \rho_{t+1}}{\partial \mu_t} = \frac{\rho_t(\lambda_t^H - \lambda_t^L)}{(\rho_t \mu_t \lambda_t^L + \lambda_t^H)^2} \geq 0.$$

We summarize the results below.

Proposition 5. *The ratio of illiterate to educated parents in the next period decreases with the current child school enrollment but increases with the current fertility differential.*

From the negative effect of school enrollment on the future ratio of illiterate to educated parents in Proposition 5, government education policy can play an important role in influencing the transition of education status across generations. The second part of Proposition 5 conforms with the conventional view: Fertility differential impedes the transition toward a better educated population. Combining these together, it is possible for a country with higher fertility differential to have a lower future ratio of illiterate to educated parents as long as it has a sufficiently higher school enrollment ratio.

From (17) and (18), the average fertility rate is determined by

$$\frac{N_{t+1}}{N_t} = \frac{n_t^H(1+\rho_t\mu_t)}{1+\rho_t}. \tag{22}$$

Equation (22) allows for demographic accounting with regard to three factors: the fertility of educated mothers n_t^H, the ratio of illiterate to educated mothers $\rho_t = N_t^L/N_t^H$, and the ratio of fertility of illiterate mothers to that of educated mothers $\mu_t = n_t^L/n_t^H$. A decline in the fertility of educated mothers or in fertility differential reduces average fertility, given any current ratio of illiterate to educated mothers. However, the effect of a lower ratio of illiterate to educated mothers on the change in average fertility over time depends on the fertility differential because sign $\partial(N_{t+1}/N_t)/\partial\rho_t$ = sign $(\mu_t - 1)$. If fertility differential μ_t exceeds 1, then a decline in ρ_t reduces average fertility over time. Conversely, if fertility differential is less than 1, then a decline in ρ_t actually raises average fertility. The result is summarized below.

Proposition 6. *If illiterate mothers have higher (lower) fertility than do educated mothers, then a decline in the ratio of illiterate to educated mothers reduces (raises) average fertility over time.*

Proposition 6 has some novel implications. First, it is the very country with illiterate mothers having higher fertility than that of educated mothers that will have a faster decline in average fertility in the transition with falling illiteracy. Conversely, a country with educated mothers having higher fertility will observe rising average fertility when illiteracy falls, unless the fertility rate of at least one type of mother falls sufficiently. Thus, it is the higher fertility differential in the usual sense that allows government education spending to achieve faster declines in not only illiteracy but also in average fertility.

The growth rate of average output from all parents in the economy is given by

$$g_t \equiv \frac{N_{t+1}^H(1 - vn_{t+1}^H)h_{t+1}^H + N_{t+1}^L(1 - vn_{t+1}^L)h_{t+1}^L}{N_t^H(1 - vn_t^H)h_t^H + N_t^L(1 - vn_t^L)h_t^L}. \tag{23}$$

Given a state $(N_t^L, N_t^H, h_t^L, h_t^H, W_t^C)$, the growth rate of average output increases if fertility rates decline over time (lower n_{t+1}^k) or if school enrollment increases (higher N_{t+1}^H). However, the growth rate from t to $t+1$ is positively associated with fertility in t, n_t^k.

From (21) and (23), the equilibrium path in the long run is given below:

Proposition 7. *For stationary $(\xi, \tau^L, \tau^H, s^L, s^H, d_t/W_t^k, W_t^C/W_t^k)$ and for logarithmic utility $\sigma = \gamma = 1$, the economy converges to a steady state growth path with the steady-*

state ratio of illiterate to educated parents

$$\rho^* = \frac{\mu(1-\lambda^L) - \lambda^H + \sqrt{[\mu(1-\lambda^L) - \lambda^H]^2 + 4\mu\lambda^L(1-\lambda^H)}}{2\mu\lambda^L} \geq 0 \tag{24}$$

and the steady-state growth rate of average output

$$g^* = A(\psi + B\xi)^\phi \left[\frac{\omega(\rho n^L\lambda^L + n^H\lambda^H) + \rho n^L(1-\lambda^L) + n^H(1-\lambda^H)}{\omega(1-vn^H) + \rho(1-vn^L)}\right]. \tag{25}$$

Proof. From (11), (14), (15) and (16) with $h_t^k = W_t^k$, the equilibrium solution for λ^k and n^k becomes stationary for $\sigma = \gamma = 1$ when $\xi_t, d_t/W_t^k$, and W_t^C/W_t^k become stationary in the long run. Consequently, $0 \leq \lambda^k \leq 1$, $\infty > \mu > 0$, and $\infty > \rho > 0$ in the long run. The steady-state ratio of illiterate to educated parents follows (21); the other root is negative and thus excluded. Consequently, the steady state is unique. Differentiating both sides of (21) with respect to ρ_t yields

$$\frac{\partial\rho_{t+1}}{\partial\rho_t} = \frac{\mu_t(\lambda_t^H - \lambda_t^L)}{(\rho_t\mu_t\lambda_t^L + \lambda_t^H)^2}. \tag{26}$$

From (26) and Proposition 3, $\lambda_t^L < \lambda_t^H$, and therefore $\partial\rho_{t+1}/\partial\rho_t > 0$. Also, from (26), the second derivative $\partial^2\rho_{t+1}/\partial\rho_t^2$ is negative under the same condition $\lambda_t^L < \lambda_t^H$ in Proposition 3. From (21), when ρ_t goes to zero, ρ_{t+1} approaches $(1-\lambda_t^H)/\lambda_t^H > 0$; when ρ_t becomes sufficiently large, the derivative in (26) falls to zero. Combining these with $\partial\rho_{t+1}/\partial\rho_t > 0$, $\partial^2\rho_{t+1}/\partial\rho_t^2 < 0$, and the uniqueness of ρ^* yields $\partial\rho_{t+1}/\partial\rho_t < 1$ at ρ^*. Such features indicate the stability of the steady state ρ^* and the globally monotonic convergence to ρ^*.

Using the transition equations of human capital in (2) and (3), the transition equations of groups of parental population in (17) and (18), and stationary $(\xi, n^k, \lambda^k, \rho)$ in the growth equation (23) yields the steady-state growth rate of average output. *Q.E.D.*

The steady state growth path applies to a special case with stationary tax and subsidy rates and with logarithmic utility. If the tax and subsidy rates increase, school enrollment ratios of all types of children tend to increase as well, leading to a lower ratio of illiterate to educated parents but a higher growth rate of average output in the long run. As noted in Greenwood et al. (2005), if $\gamma > \sigma = 1$ (fertility being less elastic intertemporally than consumption), then fertility will decline when income increases in their numerical results. We consider this situation numerically next.

2.4 *Simulation results*

To explore the quantitative implications for fertility differential, school enrollment, average fertility and economic growth, we simulate the equilibrium path with the CRRA utility numerically. Because of this utility function, fertility in the present model has no reduced form solution. The parameterization, given in Figure 2, is chosen to make our simulated fertility and school enrollment rates comparable to the available data for 50-60 years. One period of the model is regarded as 20 years. So in the first few periods we focus on the transition to capture features in the development experiences in Brazil and Indonesia. We start with higher education subsidy rates for children from illiterate mothers than for children from educated mothers (49.15% and 33.39% respectively) and increase the education subsidy rates by 10.2% per period until the school enrollment rate for children of illiterate mothers exceeds 50% or the higher subsidy rate exceeds 85%. From Table 1, government education spending as a fraction of GDP indeed increased in both countries over the 50 years. The income tax rate is 20% for educated mothers and 7% for illiterate mothers. Also, we assume $\gamma = 1.23 > \sigma = 1$, which may imply a negative trend in fertility over time.

The simulation results are reported in Figure 2. Panel (a) of Figure 2 shows that the school enrollment ratio for children of educated mothers is always higher than that for children of illiterate mothers, albeit both of them increase over time. In the first three periods (60 years), there is a significant increase in the school enrollment ratio for children of educated mothers from a level below 5% to a steady state level above 80%, whereas the enrollment for children of illiterate mothers starts from zero and rises to a lower steady state around 60%. Notice that the school enrollment curves become flat when education subsidy rates remain constant from the third period onwards. If education subsidy rates continue to increase, then the school enrollment ratios will continue to rise as well. In all periods, the tax revenue is greater than the education subsidy expenditure. The residual tax revenue is spent on G.

In Panel (b), the reported fertility rates are two times as high as the simulated fertility rates from our single gender model for a better comparison with observed fertility rates. The fertility rate of educated mothers starts higher but decreases much more rapidly than does the fertility rate of illiterate mothers, as the child school enrollment ratio of educated mothers rises first and at a higher rate. The fertility differential flips signs from a higher fertility rate of educated mothers to the opposite in the first two periods, which is consistent with the Indonesian evidence shown in the later part of the paper and the evidence documented in Vogl (2013). Note that even after school enrollment ratios converge to constant levels, the fertility rates continue to decline, although at a slower pace, because the substitution effect of higher parental wages dominates the income effect under $\gamma > \sigma = 1$ as noted in Greenwood et al. (2005). If logarithmic utility is used instead, then fertility rates will converge to constant levels when school enrollment ratios become constant.

Panel (c) plots the growth rate of average output and the average fertility rate of all parents. The average fertility rate declines from a level above three to the replacement level of two in period four. After period four, average fertility declines further at a slower pace. The simulated growth rate displays large swings because of the uneven paces of changes in school enrollment ratios and fertility rates over time and across different groups of families. For instance, when the school enrollment ratios of all children increase rapidly to their steady state level in period 3, the growth rate of average output rises sharply to its peak level of 4% in period 4. This result is novel compared to the conventional case with smooth and gradual changes in growth rates under the assumption of continuously divisible levels of education.

In Panel (d), the simulated share of educated mothers in the parental generation starts near zero, changes little in the first two periods because of the opposing effects of rising school enrollment but falling fertility from educated mothers. It jumps up rapidly in periods three and six due to rising school enrollment ratios for all children over 60%. Afterwards, the ratio of educated mothers to the parental generation continues to rise gradually to exceed 80%.

We have also run many rounds of experiments with different parameterizations, especially alternative education subsidy rates. Increasing the education subsidy rate is important to influence the pace of development featuring human capital accumulation and the demographic transition. As mothers choose higher school enrollment ratios for children from generation to generation, the downward intergenerational mobility for children of educated mothers is falling and at the same time the upward intergenerational mobility for children of illiterate mothers is rising. The diverse intergenerational mobilities of children of a same mother here is through the mother's rational choice instead of exogenous or random chances in the literature, such as Fan and Zhang (2013).

Overall, the simulated equilibrium path captures the levels and patterns of movements of the concerned variables observed in Brazil and Indonesia in Table 1 and Figure 1. For more details about the observations in these countries, we now turn to empirical evidence based on census data.

3 Empirical evidence

Our empirical analysis focuses on Brazil and Indonesia for the following reasons. First, both countries have the longest sample coverage among developing countries, which enables us to observe the transition with declining fertility rates and rising school enrollment ratios. Second, Brazil has one of the largest fertility differential in the world but Indonesia has one of the lowest fertility differential, as pointed out in Kremer and Chen (2002). A comparison of the demographic transition and school enrollment between these two countries should thus help us to shed more lights on the distribution and evolution of fertility and school enrollment across mother groups and over time. In particular, this comparison can enhance the understanding of how fertility differential interacts with school enrollment during the demographic transition and development.

3.1 *The data*

The data used in this paper are extracted from the Integrated Public Use Microdata Series (IPUMS). For Brazil, we use the 5% sample of the 1960, 1970, and 1980 censuses, the 5.8% sample of the 1991 census, the 6% sample of the 2000 census, and the 5% sample of the 2010 census. For Indonesia, we use the 0.54% sample of the 1971 census, the 0.22% sample of the 1976 census, the 5% sample of the 1980 census, the 0.37% sample of the 1985 census, the 0.51% sample of the 1990 census, the 0.43% sample of the 1995 census, and the 10% sample of the 2000 and 2010 censuses as fertility information was not collected in the 2005 Indonesia census.

In dealing with data about the demographic transition, we use *actual* fertility at the survey time, instead of the *total* fertility rate (TFR) which is an averaged age-specific birth rate of all childbearing age women. The advantage of using actual fertility is that it reveals the decision of a particular group of women. On the other hand, the TFR is a synthetic rate affected by both the fertility decision and the age composition of the interested population. During a period with considerable changes in fertility, there could be a sizable difference between the TFR and the actual fertility rate of any particular group. For instance, if the fertility rate declined at the same pace across all birth cohorts, then the TFR would overestimate the actual fertility rate of the current young generation as they would have lower fertility than the current older generations at any given age.

We restrict our sample to women aged 40 to 49 in the census year. The reason for doing so is that the number of children born over a woman's lifetime can only be accurately constructed for those who were at the end of their reproductive cycle. As a result, we have observations on fertility and child school enrollment of six birth cohorts in Brazil and seven in Indonesia. Women with missing fertility information are excluded.[4] Because we can only match mothers with their children if they lived in the same household, we focus on the education outcomes of children aged between 7 and 15 to avoid our child sample being over-represented by children who left home at older

[4]We have also used the number of survival children rather than the number of live births as the measure for fertility. All of our conclusions are insensitive to which measure is used in the analysis.

ages. As many children in this age group were still enrolled in school, we use school enrollment status rather than years of schooling to measure their educational outcome.

Table 1 reports the descriptive statistics of the key variables.[5] According to these statistics, both countries continued their demographic transition process for the 50 years of the sample period to the extent that took about 100 years to accomplish in most developed countries. These two countries had similar average fertility rates and shared comparable fertility trends. For example, on average, the sampled women born in the 1920s had 5.5 children in Brazil and 5.2 children in Indonesia. The respective numbers declined to 2.4 and 3.0 for women born in the 1960s in these two countries. The gap between the average fertility in column (1) of Table 1 and TFR in column (2) arises from the fact that the fertility of younger women below age 40 was excluded from the former for aforementioned reasons.

Women's educational attainment and their children's school enrollment ratios also increased considerably over time. Women's schooling years increased at a slightly slower pace but child school enrollment increased at a faster pace in Brazil than in Indonesia. Real per capita GDP in Brazil was more than twice as that in Indonesia and increased substantially in both countries at a faster pace in Indonesia. The 10-year average growth rates peaked first at 6% in 1970-1980 and again at 2% and 3.7% respectively in 2000-2010 in Brazil and Indonesia, following significant increases in school enrollment ratios in the preceding decades. The ratio of public education spending to GNI in Indonesia was below 1% before 2000, which is much lower than the 4% level observed in Brazil throughout the sample period. But at the end of the sample period, public education spending as percent of GNI in Indonesia was comparable to that in Brazil.

3.2 *School enrollment*

To check the validity of our result that parents enroll a fraction of their children to school, we need to calculate the school enrollment ratio. Because school enrollment

[5]The GDP series is extracted from the Penn World Table 7.1. The ratio of education expenditure to GNI is extracted from the World Development Indicator.

ratios vary with age, the existence of within sibling variations in school enrollment at any given year does not necessarily imply that parents only enroll a fraction of their children after controlling for children's age. To minimize the impact of age on school enrollment, we first focus on children aged between 13 and 15. These children were not only old enough to leave school in developing countries, but also young enough such that their school enrollment was still mostly their parents' decision. In contrast, the school enrollment ratio of primary school age children heavily depends on a country's compulsory education law rather than on their parents.

Even within this narrowly defined age group, the school enrollment ratio of 13-year-olds is much higher than that of 15-year-olds. Therefore, we focus on the conditional school enrollment ratio of the youngest child from households with multiple children who fell into the selected age group, rather than the proportion of children enrolled in school in the census year. In particular, we report the school enrollment ratio of the youngest children conditional on the school enrollment status of their oldest siblings. For comparison purpose, we also calculate the unconditional school enrollment ratio.

Column (1) of Table 2 reports the unconditional school enrollment ratio of the youngest child, $P(S_y = 1)$, where $S_y = 1$ if the youngest child is enrolled in school and 0 otherwise. This unconditional school enrollment ratio $P(S_y = 1)$ increased from 44.1% in 1960 to 95.7% in 2010 in Brazil and from 76.3% in 1976 to 89.6% in 2010 in Indonesia. Column (2) reports the conditional school enrollment ratio $P(S_y = 1|S_o = 1)$, where $S_o = 1$ if the oldest child had been enrolled in school and 0 otherwise. It should be noted that this ratio can only serve as an upper bound for the enrollment variable λ even among families who indeed sent at least one child to school. First, as documented by Parish and Willis (1993), younger children from poor families tend to have more years of schooling than their older siblings. Second, the younger children will have a higher school enrollment ratios due to the fast growth in school enrollment during the sample period. Even as an upper bound, we still observe that some families only enrolled some of their children to school, particularly in the early sample period. The evidence provides empirical support for our departure from the conventional approach of choosing an equal amount of education to all siblings. This conditional enrollment ratio hovered around 91% in Brazil and 93% in Indonesia from 1970 to 1990. Since then, this conditional enrollment ratio jumped in both countries. For comparison purpose, we also report $P(S_y = 1|S_o = 0)$. Because children enrolled in school in the census year might dropped out of school later, the youngest children enrolled in school from families with $S_y = 1$ and $S_o = 0$ may not necessarily have more years of schooling than their older siblings eventually. However, as long as some of these younger children stayed at school for two more years, these who stayed at school would have more education than their older siblings. Moreover, the steady upward trend in $P(S_y = 1|S_o = 0)$ does suggest that the probability of sending at least one child to school increased over time in both Brazil and Indonesia.

Figure 1 plots school enrollment ratios of children aged 7–15 by mothers' educational attainment. We split the sample into four groups by mothers' educational attainment: illiterate, some primary school but uncompleted, completed primary school, and completed secondary school. Overall, better educated mothers had higher child school enrollment in both countries, as predicted in Proposition 3. For children of the most educated and least educated women, their school enrollment ratios were comparable between Brazil and Indonesia in the 1970s. But the school enrollment ratio was lower for Indonesian children born to mothers with some or completed primary school education than for their Brazilian counterparts. Moreover, the child school enrollment of illiterate women increased at a much faster pace in Brazil despite a much larger fertility differential. This could arise from the much higher government education spending in Brazil than in Indonesia from Table 1. Nevertheless, the evidence suggests that a higher fertility differential does not necessarily imply slower human capital accumulation during the demographic transition process.

It should be noted that the upward trend revealed in Figure 1 might be biased by changes in the age composition of the sampled children. Because we restricted our sample to mothers aged between 40 and 49 in the census year and mothers' age at childbirth might have increased with their education level over time, the average age of our sampled children could be systematically different across census years. Since many children dropped out of school after completing primary school, changes in the age profile of the sampled children could cause year to year variations in school enrollment even if the age-specific school enrollment ratio does not change over time. To control for the impact of changes in the age profile of our sampled children, we run the following logit regression

$$P(E_{it} = 1) = F(\gamma_0 + \gamma_1 M_i + \sum_{a=8}^{a=15} \gamma_a D_{ai} + \sum_{j=1}^{j=4} \sum_{t} Z_j C_t \delta_{jt}), \tag{27}$$

where $E_{it} = 1$ if child i is enrolled in school at time t, $M_i = 1$ if child i is a male, D_a is a series of age dummies, Z_j is a series of mothers' educational attainment dummies, and C_t is a series of census year dummies.

Table 3 reports the estimation results. Columns (1)-(4) of Table 3 are the coefficients on the interaction between the mother's education dummy and the census year dummy, $\hat{\delta}_{jt}$, whereas columns (5)-(8) are the marginal effects of these interaction terms. For the sake of brevity, the coefficients on the gender dummy and age dummies are not reported. We use children born to illiterate mothers in the 1960 Brazilian census (1971 Indonesian census) as the reference group. Consequently, the marginal effect of the interaction term captures the difference in the school enrollment ratio between children in the reference group and children in a concerned census year whose mother belonged to the given education group, when evaluated at the sample mean. The vertical reading of the table reveals the time trend of school enrollment for a given level of maternal education, while the horizontal reading shows the impact of maternal education on their children's school enrollment at any given time. In both countries, maternal education has a strong positive effect on their children's school enrollment ratios.

For instance, in Brazil, the child school enrollment ratio of mothers with a secondary school education is 54.7 percentage points higher than that of illiterate mothers in 1960. The difference in child school enrollment ratios across mothers' education levels in Indonesia is smaller than that in Brazil. For instance, in 1980 when the school enrollment ratio information is available for both countries, the child enrollment ratio of mothers with at least secondary education is 33.6 percentage points higher than that of illiterate mothers in Indonesia. In comparison, the difference in Brazil is 59.7 percentage points. This evidence supports the theoretical results in Proposition 3: Better educated mothers have higher school enrollment ratios for children.

The estimation results also show that school enrollment increased over time regardless of maternal education even after controlling for child age. For instance, comparing children surveyed in the 1960 and 2010 Brazilian censuses, the school enrollment ratio increased by 47.1 percentage points among children of illiterate mothers and by 11 percentage points among children of middle school graduates. As a result, the difference in child school enrollment ratios between illiterate mothers and mothers with at least secondary school education narrowed from 54.7 percentage points in 1960 to 18.7 percentage points in 2010.

The time trend of school enrollment ratios in Brazil was non-monotonic: the school enrollment in the 1980 census was lower than that in the 1970 census. Meanwhile, there was substantial growth in per capita GDP in Brazil (from $3844.6 in 1970 to $6943.1 in 1980), preceded by a surge in school enrollment during 1960-1970. According to equation (11), if wages grow faster than doe the living cost of children, then school enrollment can fall. After a decade of school enrollment decline, per capita GDP decreased significantly from $6943.1 in 1980 to $6162.6 in 1991, accompanied by the rise in the child school enrollment of mothers who never attended or never completed primary school education. From (11), if wages grow at a lower pace than the living cost of children, then school enrollment can rise.

In Indonesia, like in Brazil, the increase in school enrollment ratio is also not monotonic. While the child school enrollment of illiterate mothers kept increasing throughout the entire sample period, the increase was not as smooth for other groups. It declined slightly from 1976 to 1980 for children of middle school graduates, from 1980 to 1990 for children whose mother had some formal education but never graduated from middle school, and from 1995 to 2010 for children whose mothers had graduated from primary or middle school. The school enrollment decline during the 1970-1990 period was accompanied by the most rapid growth in per capita GDP, as in Brazil.

The faster increase in the school enrollment ratio of children whose mother never attended or never completed primary school in Brazil is consistence with the transition mechanics described in Section 2: with the help of proper government policies, countries with a higher fertility differential could even enjoy faster human capital growth in the transition process.[6]

As the child school enrollment ratio increases over time, the population share of less educated mothers decreases. Table 4 reports the population share of mothers by educational attainment. The population share of the least educated mothers decreased considerably in both countries, from 47.2% for the 1920s cohort to 0.9% for the 1960s cohort in Brazil and from 79.5% to 10.2% in Indonesia.

3.3 *Fertility differential and the demographic transition*

Following de la Croix and Doepke (2003), we use the ratio of the fertility rate of the least educated group to that of the most educated group as the measure for fertility differential.

Table 5 reports fertility differential and fertility rates by maternal education. As documented by Kremer and Chen (2002) and de la Croix and Doepke (2003), fertility differential was much larger in Brazil than in Indonesia. For the 1920s cohort, the fertility rate of illiterate women was 2.5 times as large as that of middle school graduates in Brazil but was about 10% lower than that of middle school graduates in Indonesia. While fertility differential in Brazil is in line with the Beckerian theory that emphasizes the *substitution effect* of maternal education on fertility, the opposite fertility differential in Indonesia appears to be in line with the Malthusian theory that emphasizes the *income effect* when taking a positive relation between income and education for granted. The difference between these two countries is likely due to the interactions between parental

[6] While the Brazilian government spent 3.599% of its gross national income on education in 1980, the corresponding figure in Indonesian was only 0.767%.

wage and child living cost as pointed out in Proposition 4. Since per capita GDP in Indonesia was less than 1/3 than that in Brazil during 1960-1980, it is likely that the child living cost in Indonesia during that period was relatively large compared to the child wage and education cost. Consequently, while the income effect of parental wage dominates the substitution effect on fertility in Indonesia, the opposite is true in Brazil.

From (22), both group specific fertility and the population share of mothers of different education group affect the average fertility. To explore the contributions of changes in education composition and in the group specific fertility rates to the overall average fertility decline, we conduct an exercise of average fertility accounting over three generations, 20 years per generation, across four groups of mothers by education for both countries. The accounting procedure is to allow for variation in either the respective fertility rates in each group of mothers (columns (2)-(5) of Table 5) or the shares of mothers by education (columns (1)-(4) of Table 4), one at a time. In essence, the procedure is an extension of the equilibrium framework in Section 2.3 to consider more groups of mothers.

According to Table 1, the average fertility rate in Brazil was 5.481 in 1970, 4.105 in 1991, and 2.405 in 2010, declining by 59% over three generations. Holding the group fertility rates at the levels in 1970, the variation in the shares of mothers by education reduces average fertility to 4.696 in 1991 and to 3.461 in 2010. That is, the variation in mothers' education composition alone will reduce the average fertility rate by 2.02, which can account for 65.7% of the fertility decline. On the other hand, the variation in group fertility rates alone reduces average fertility to 4.832 in 1991 and to 3.193 in 2010, which accounts for 74.4% of the decline in average fertility. In this sense, the overall rise in mothers' education status played an almost equally important role in the demographic transition in Brazil as did the decline in group specific fertility rates. From Table 4, the most dramatic change was the large decline in the share of illiterate mothers (from 47.2% in 1970 to 24.0% in 1991 and to 0.9% in 2000) and the corresponding increases in the shares of educated mothers (completing primary and secondary education in particular). This is consistent with the substantial increases in school enrollment ratios in Figure

1A in Brazil. Therefore, the large increase in school enrollment was one of the major driving forces for the demographic transition in Brazil. Such results are consistent with the predictions in Proposition 6.

In Indonesia, by contrast, the decline in average fertility went mainly through the decline in fertility rates of mothers in all groups despite a similar dramatic decline in the share of illiterate mothers and an increase in the share of mothers completing middle school. Because illiterate mothers had lower fertility rates than did educated mothers in Indonesia initially, a similar change in mothers' education profile counteracted the declining trend of average fertility to some extent. The actual levels of average fertility of Indonesia were 5.227 in 1971, 4.793 in 1990, and 2.992 in 2010. Given that educated mothers had higher fertility than did illiterate mothers, the variation in mothers' shares by education status alone would predict a rising trend in average fertility: 5.636 in 1990 and 5.833 in 2010 as predicted by Proposition 6. The rise in average fertility was indeed observed from 5.229 in 1971 to 5.392 in 1980. On the other hand, the variation in group fertility rates alone predicts a declining trend: 4.673 in 1990 and 3.196 in 2010, which explains 90% of the actual changes. Such fertility declines for all groups of mothers in Indonesia could arise in a scenario where the negative substitution effect of higher maternal educational attainment and higher wages on fertility dominated the positive income effect as discussed earlier.

4 Conclusion

In this paper, we have attempted, theoretically as well as empirically, to understand differential fertility and differential child school enrollment ratios among mothers with different education status in the demographic transition and economic development. Some theoretical contributions are made from an overlapping-generations model in which parents with different education levels choose the number of children and the fraction of children for school education. The remaining fraction of children work as child labor for supplemental income to their parents. The fraction of children for school education is increasing not only with the wage premium, as conventionally thought, but also with the costs of time and income for rearing a child; and it is decreasing with the potential child labor earning. Also, better educated parents have higher child school enrollment ratios. The crucial factor for the school enrollment differential is the difference in the forgone wage for rearing a child.

However, better educated parents may have higher or lower fertility because of the opposite income and substitution effects of higher parental wage. When wages and education costs are low enough relative to the child living cost in early development, fertility is higher for better educated parents, and the increase in parental wages increases fertility as observed in the early stage of the demographic transition. In a later stage of development, wages and education costs are high enough relative to the child living cost, fertility becomes higher for less educated parents, and the increase in wages reduces fertility, first for educated parents and later for all types of parents. This causes fertility differential to flip signs from better educated mothers having more children to the opposite.

At the aggregate level, the ratio of illiterate to educated mothers converges monotonically to a unique steady state regardless of the starting value, ensured by the above result that better educated parents choose a higher school enrollment ratio for their children. Moreover, higher child school enrollment, regardless of parental education status, reduces the future ratio of illiterate to educated parents, whereas higher fertility differential raises the future ratio of illiterate to educated mothers. Finally, a decline in the ratio of illiterate to educated mothers in the transition process reduces average fertility if illiterate mothers have higher fertility, but raises average fertility if illiterate mothers have lower fertility. While the prediction of the mechanics is consistent with the conventional wisdom that a greater fertility differential implies a lower level of human capital in the steady state, it also yields a surprising implication: A greater fertility differential coupled with higher child school enrollment could even yield a steeper decline in fertility and a larger rise in the level of human capital. The latter implication justifies the role of government education policy in accelerating the transition process against the inertia of fertility differential.

Our simulation results capture rapid increases in school enrollment ratios and significant declines in fertility, first from educated mothers who initially have higher fertility than illiterate mothers do. During this process, there is a significant decline in average fertility, a switch of signs of fertility differential, and two waves of high growth rates of per worker output, as school enrollment ratios increase for the two groups of children with a lag. The simulation results also confirm a conducive role of education subsidization in the development process through promoting school enrollment and reducing fertility.

Using data extracted from six Brazilian population censuses and seven Indonesian population censuses, we obtain some challenging observations to the conventional wisdom. Despite their very different fertility differentials in the beginning of the sample period, both countries shared the same declining trend in fertility and same rising trend in school enrollment. Actually, the child school enrollment ratio of illiterate mothers rose at a faster pace in Brazil than in Indonesia. Thus, human capital could grow faster with a high fertility differential, at least when government education spending was much greater as shown in our simulation. Indeed, government education spending as percent of GDP was much higher in Brazil than in Indonesia until the latter catches up very recently. While such observations pose questions about the conventional wisdom about the role of fertility differential in the transition, they are consistent with the predictions of the mechanics described in this paper.

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Table 1: Descriptive Statistics

Census Year	Fertility	TFR	Mother's Schooling	Child School Enrollment	Real GDP per capita	Education Exp. (% of GNI)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Brazil						
1960 (born in 1911-1920)	5.916	6.21	1.538	.482	2469.1	-
	(4.517)		(2.261)	(.500)		
1970 (born in 1921-1930)	5.481	5.02	2.115	.660	3844.6	3.599
	(4.187)		(2.844)	(.474)		
1980 (born in 1931-1940)	5.124	4.07	2.998	.658	6943.1	3.599
	(3.801)		(3.481)	(.475)		
1991 (born in 1942-1951)	4.105	2.72	4.553	.759	6116.8	4.634
	(3.183)		(4.425)	(.428)		
2000 (born in 1951-1960)	3.141	2.36	6.276	.935	6834.3	3.881
	(2.444)		(4.619)	(.247)		
2010 (born in 1961-1970)	2.405	1.80	8.084	.963	8324.7	4.328
	(1.836)		(4.578)	(.189)		
Panel B: Indonesia						
1960 (born in 1911-1920)	-	5.67	-	-	665.15	-
1971 (born in 1922-1931)	5.227	5.41	.909	.577	873.7	0.767
	(3.277)		(2.023)	(.494)		
1976 (born in 1927-1936)	5.268	4.93	1.296	.656	1239.9	0.767
	(3.308)		(2.385)	(.475)		
1980 (born in 1931-1940)	5.392	4.43	1.725	.766	1496.5	0.767
	(3.254)		(2.816)	(.423)		
1990 (born in 1941-1950)	4.793	3.12	3.638	.809	2160.1	0.626
	(2.677)		(3.687)	(.393)		
1995 (born in 1946-1955)	4.130	2.70	4.963	.873	2882.6	0.610
	(2.337)		(3.774)	(.333)		
2000 (born in 1951-1960)	3.790	2.45	5.722	-	2749.6	2.228
	(2.307)		(3.422)			
2010 (born in 1961-1970)	2.992	2.10	7.194	.914	3965.8	4.328
	(1.875)		(4.100)	(.281)		

Notes: The fertility rate and mother's schooling are calculated using a sample of women aged 40-49 at the census year.
Enrollment is calculated using 7-15-year old children whose mother aged between 40-49 in the census year.
School enrollment information was not collected in the 2000 Indonesian population census.
Numbers in parentheses are standard deviations.
Real GDP per capita is valued at the 2005 constant prices, extracted from the Penn World Table version 7.1.
The Total Fertility Rate (TFR) and Education Expenditure are extracted from the World Development Indicator. Education Expenditure only includes public expenditure.

Table 2: Child school enrollment ratios by older sibling's enrollment status

Census year	All	Older sibling enrolled		Older sibling not enrolled	
	$P(S_y = 1)$	$P(S_y = 1 \mid S_o = 1)$	Observations	$P(S_y = 1 \mid S_o = 0)$	Observations
	(1)	(2)	(3)	(4)	(5)
Panel A: Brazil					
1960	.441	.884	4,078	.266	10,278
1970	.652	.912	13,891	.374	13,072
1980	.672	.910	17,309	.355	13,037
1991	.702	.908	20,621	.397	13,943
2000	.915	.961	25,869	.651	4,576
2010	.957	.982	17,044	.654	1,421
Panel B: Indonesia					
1971	.763	.938	980	.487	622
1976	.752	.936	531	.512	408
1980	.778	.926	12,287	.570	8,748
1990	.791	.933	1,614	.560	988
1995	.892	.976	1,656	.637	543
2010	.896	.959	28,534	.648	7,322

Notes: The sample consists of children aged 13-15 with at least a sibling in the same age range and whose mother aged 40-49 in the census year. Because the 2000 Indonesia census does not contains information on school enrollment, we cannot calculate the school enrollment ratio for that year.

$P(S_y = 1)$ is the school enrollment ratio of the youngest child in a household. $P(S_y = 1 \mid S_o = 1)$ is the school enrollment ratio of the youngest child in a household conditional on the oldest child enrolled in school. $P(S_y = 1 \mid S_o = 0)$ is the school enrollment ratio of the youngest child in a household whose oldest child not enrolled in school.

Table 3: The impact of maternal education on child school enrollment

Census year	Coefficients				Marginal effect			
	E_1	E_2	E_3	E_4	E_1	E_2	E_3	E_4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Brazil								
1960 (mother born in 1911-1920)		1.408	3.603	4.131		.186	.477	.547
		(.015)	(.086)	(.178)		(.002)	(.011)	(.023)
1970 (mother born in 1921-1930)	.703	2.099	4.126	5.107	.093	.278	.546	.676
	(.012)	(.013)	(.076)	(.101)	(.002)	(.002)	(.01)	(.013)
1980 (mother born in 1931-1940)	.586	1.991	3.564	4.512	.078	.263	.471	.597
	(.012)	(.012)	(.046)	(.062)	(.002)	(.002)	(.006)	(.008)
1991 (mother born in 1942-1951)	.866	2.053	3.236	4.067	.115	.272	.428	.538
	(.012)	(.011)	(.027)	(.027)	(.002)	(.001)	(.004)	(.004)
2000 (mother born in 1951-1960)	2.452	3.286	4.159	5.119	.324	.435	.55	.677
	(.015)	(.013)	(.027)	(.031)	(.002)	(.002)	(.004)	(.004)
2010 (mother born in 1961-1970)	3.557	3.853	4.305	4.974	.471	.51	.569	.658
	(.076)	(.017)	(.026)	(.027)	(.01)	(.002)	(.003)	(.004)
Number of observations	2,209,392							
Panel B: Indonesia								
1971 (mother born in 1922-1931)	-	1.075	1.872	3.119	-	.105	.183	.304
	-	(.038)	(.045)	(.104)	-	(.004)	(.004)	(.010)
1976 (mother born in 1927-1936)	.216	1.194	2.162	3.698	.021	.116	.211	.361
	(.028)	(.041)	(.071)	(.173)	(.003)	(.004)	(.007)	(.017)
1980 (mother born in 1931-1940)	.659	1.377	2.211	3.448	.064	.134	.216	.336
	(.017)	(.018)	(.022)	(.036)	(.002)	(.002)	(.002)	(.004)
1990 (mother born in 1941-1950)	.827	1.285	2.061	3.622	.081	.125	.201	.353
	(.025)	(.026)	(.033)	(.074)	(.002)	(.003)	(.003)	(.007)
1995 (mother born in 1946-1955)	.959	1.720	2.543	4.522	.094	.168	.248	.441
	(.032)	(.032)	(.037)	(.098)	(.003)	(.003)	(.004)	(.010)
2010 (mother born in 1961-1970)	1.282	1.772	2.246	3.465	.125	.173	.219	.338
	(.018)	(.018)	(.017)	(.018)	(.002)	(.002)	(.002)	(.002)
Number of observations	1,816,201							

Notes: The sample consists of children aged 7-15 and whose mother was aged 40-49 at the census year.
E_1 refers to illiterate women, E_2 to refers women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school.
Numbers in the parentheses are standard errors, which are clustered at the household level.
School enrollment information was not available in the 2000 Indonesia census.
Columns (1)-(4) report the coefficients on the interaction between mother's education and census year dummy from the logit regression and columns (5)-(6) report the marginal effects. In addition to the interaction term, child gender and age are included as a vector of dummy variables in the regression.

Table 4: Population shares by maternal educational attainment

Census year	E_1 (1)	E_2 (2)	E_3 (3)	E_4 (4)
	Panel A: Brazil			
1960 (born in 1911-1920)	.566	.387	.035	.012
1970 (born in 1921-1930)	.472	.445	.041	.042
1980 (born in 1931-1940)	.368	.489	.064	.079
1991 (born in 1942-1951)	.240	.487	.092	.180
2000 (born in 1951-1960)	.114	.434	.155	.298
2010 (born in 1961-1970)	.009	.330	.215	.447
	Panel B: Indonesia			
1971 (born in 1922-1931)	0.795	0.118	0.072	0.014
1976 (born in 1927-1936)	0.691	0.217	0.065	0.026
1980 (born in 1931-1940)	0.626	0.241	0.089	0.044
1990 (born in 1941-1950)	0.360	0.290	0.229	0.121
1995 (born in 1946-1955)	0.198	0.304	0.313	0.185
2000 (born in 1951-1960)	-	0.337	0.454	0.209
2010 (born in 1961-1970)	0.102	0.098	0.441	0.360

Notes: The sample consists of women aged 40-49 at the census year.
E_1 refers to illiterate women, E_2 refers to women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school.
Illiterate women and these who did not graduate from primary school were grouped together in the 2000 Indonesia census.

Table 5: Fertility rates by maternal education

Census year	μ (1)	E_1 (2)	E_2 (3)	E_3 (4)	E_4 (5)
	Panel A: Brazil				
1960 (born in 1911-1920)	2.742	6.663	5.205	2.845	2.430
1970 (born in 1921-1930)	2.555	6.428	4.991	2.929	2.515
1980 (born in 1931-1940)	2.572	6.355	4.889	3.104	2.471
1991 (born in 1942-1951)	2.664	5.852	4.177	2.903	2.197
2000 (born in 1951-1960)	2.540	5.012	3.618	2.678	1.973
2010 (born in 1961-1970)	1.967	3.516	3.050	2.410	1.787
	Panel B: Indonesia				
1971 (born in 1922-1931)	0.901	5.019	5.995	6.188	5.568
1976 (born in 1927-1936)	0.960	5.038	5.784	6.001	5.249
1980 (born in 1931-1940)	0.952	5.101	5.939	5.967	5.358
1990 (born in 1941-1950)	1.062	4.596	5.088	4.977	4.329
1995 (born in 1946-1955)	1.134	4.062	4.393	4.242	3.583
2000 (born in 1951-1960)	-	-	4.038	3.830	3.303
2010 (born in 1961-1970)	1.198	3.181	3.434	3.125	2.655

Notes: The sample consists of women aged 40-49 at the census year.
μ is the measure of fertility differential, defined the as the ratio of illiterate women' fertility to that of secondary school graduates.
E_1 refers to illiterate women, E_2 refers to women with some primary school education, E_3 refers to women who completed primary school, and E_4 refers to women who graduated from secondary school.
Illiterate women and those who did not graduate from primary school were grouped together in the 2000 Indonesia census.

Figure 1: Child school enrollment rate by maternal education

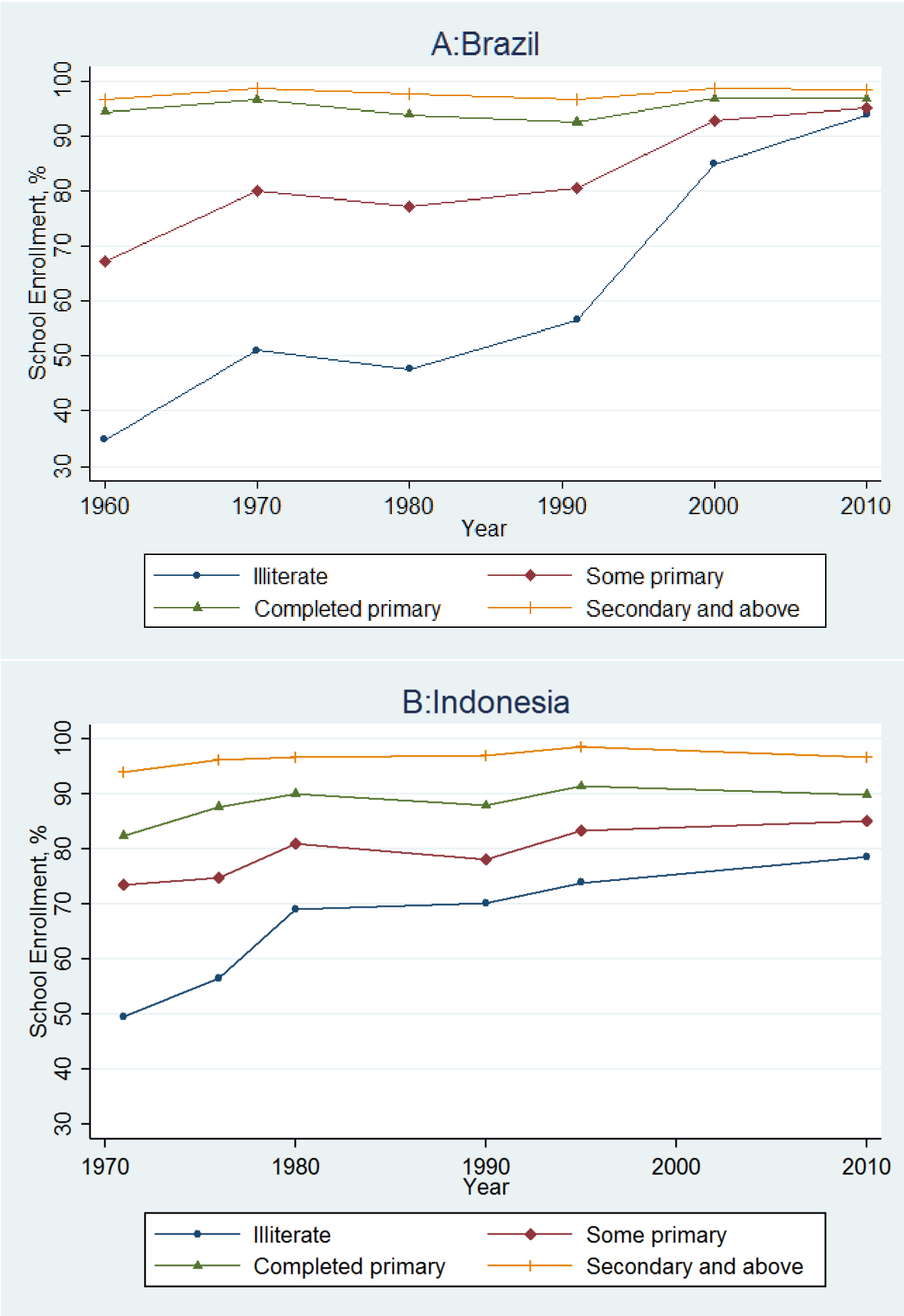
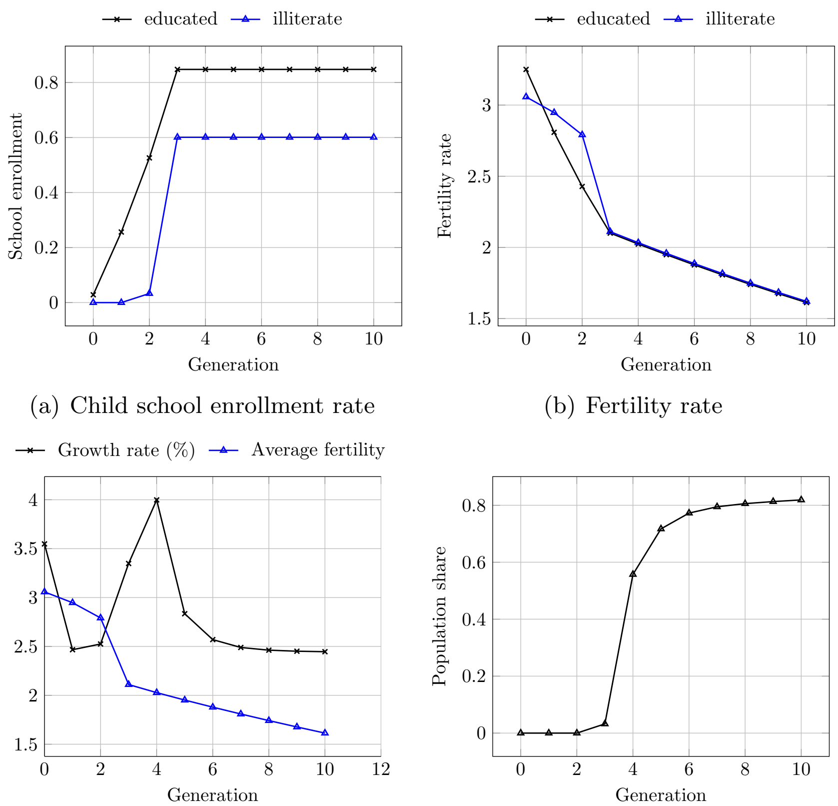


Figure 2: Fertility, school enrollment and economic growth



(a) Child school enrollment rate

(b) Fertility rate

(c) Growth rate and average fertility

(d) Share of educated workers

Parameterization: $\alpha = 0.56, \beta = 0.3, \eta = 0.44, \gamma = 1.23, \sigma = 1, \upsilon = 0.06, \xi = 0.03,$ $\phi = 0.9, \psi = 0.1, A = 7.5, B = 2.6, \tau^H = 0.2, \tau^L = 0.07,$ $\theta = 1.102$ if $\lambda_t^L < 0.5$ and 1 otherwise, $s_t^H = 0.3339\theta^t, s_t^L = 0.4915\theta^t,$ $H_0^H = 3.3754, H_0^L = 1.8884, d_t/W_t^L = 0.04,$ and $W^C/W_t^L = 0.02$.